Exemplar Problem with Solution :

Q)

If $A + B + C = 0$, then prove that $\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix} = 0.$

Soln :

$$\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix}$$

$= 1(1 - \cos^2 A) - \cos C(\cos C - \cos A \cdot \cos B) + \cos B(\cos C \cdot \cos A - \cos B)$

$= \sin^2 A - \cos^2 C + \cos A \cdot \cos B \cdot \cos C + \cos A \cdot \cos B \cdot \cos C - \cos^2 B$

$= \sin^2 A - \cos^2 B + 2\cos A \cdot \cos B \cdot \cos C - \cos^2 C$

$= -\cos(A + B) \cdot \cos(A - B) + 2\cos A \cdot \cos B \cdot \cos C - \cos^2 C$

$[\because \cos^2 B - \sin^2 A = \cos(A + B) \cdot \cos(A - B)]$

$= -\cos(-C) \cdot \cos(A - B) + \cos C(2\cos A \cdot \cos B - \cos C)$

$= -\cos C(\cos A \cdot \cos B + \sin A \cdot \sin B - 2\cos A \cdot \cos B + \cos C)$

$= \cos C(\cos A \cdot \cos B - \sin A \cdot \sin B - \cos C)$

$= \cos C[\cos(A + B) - \cos C]$

$= \cos C(\cos C - \cos C)$ (As $\cos C = \cos(A + B)$)

$= 0$