Exemplar Problem with Solution :

Using the properties of Determinants prove the following :

$$\begin{vmatrix} z^2x^2 & zx & z+x \\ x^2y^2 & xy & x+y \end{vmatrix} = 0$$

$$\begin{vmatrix} y^2z^2 & yz & y+z \\ z^2x^2 & zx & z+x \\ x^2y^2 & xy & x+y \end{vmatrix}$$

[Multiplying R_1, R_2, R_3 by x, y, z respectively]

$$= \frac{1}{xyz}\begin{vmatrix} xy^2z^2 & xyz & xy+xz \\ x^2yz^2 & xyz & yz+xy \\ x^2y^2z & xyz & xz+yz \end{vmatrix}$$

[Taking (xyz) common from C_1 and C_2]

$$= \frac{1}{xyz}(xyz)^2\begin{vmatrix} yz & 1 & xy+xz \\ xz & 1 & yz+xy \\ xy & 1 & xz+yz \end{vmatrix}$$

[Applying $C_3 \to C_3 + C_1$]

$$= xyz\begin{vmatrix} yz & 1 & xy+yz+zx \\ xz & 1 & xy+yz+zx \\ xy & 1 & xy+yz+zx \end{vmatrix}$$

[Taking $(xy+yz+zx)$ common from C_3]

$$= xyz\,(xy+yz+zx)\begin{vmatrix} yz & 1 & 1 \\ xz & 1 & 1 \\ xy & 1 & 1 \end{vmatrix}$$

$= 0$ [$\because$ C_2 and C_3 are identical]