Q) Evaluate $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$

Let $x = a\tan^2\theta \Rightarrow dx = 2a\tan\theta\sec^2\theta\,d\theta$

$\therefore \int \sin^{-1}\sqrt{\sin^2\theta}\cdot 2a\sec^2\theta\tan\theta\,d\theta$

$= 2a\int \theta\sec^2\theta\tan\theta\,d\theta$

Using by parts

$= 2a\left[\frac{\theta\tan^2\theta}{2} - \int\frac{\tan^2\theta}{2}\,d\theta\right]$

$= a\left[\theta\tan^2\theta - \int(\sec^2\theta - 1)\,d\theta\right]$

$= a\left[\theta\tan^2\theta - \tan\theta + \theta\right] + c$

Now put $\theta = \tan^{-1}\sqrt{\frac{x}{a}}$

$= a\left[\frac{x}{a}\cdot\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + \tan^{-1}\sqrt{\frac{x}{a}}\right] + \theta c$