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NOTES - Parabola

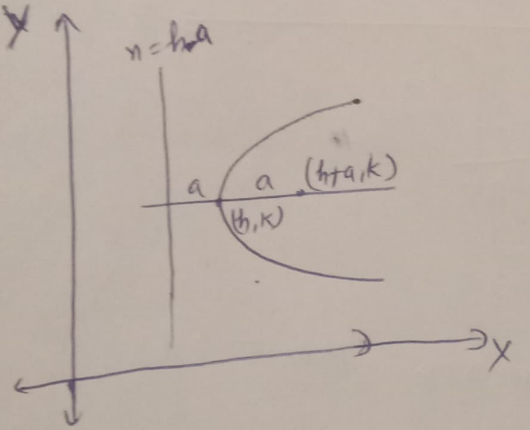
- Parabola :- Locus of a point that moves on a plane such that its distance from a fixed point is always equal to its distance from a fixed line.

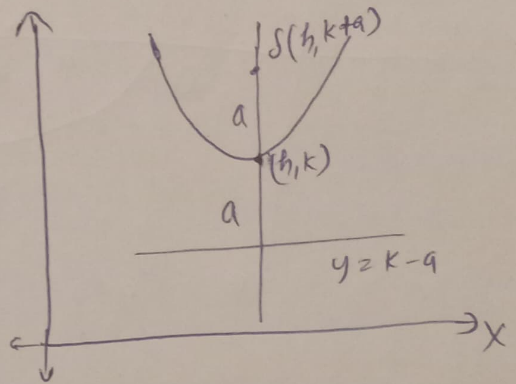
- Equation of Parabola :-

  → Standard Equation of Parabola -

$$(y-k)^2 = 4a(x-h)$$

$$\text{or } (x-h)^2 = 4a(y-k)$$



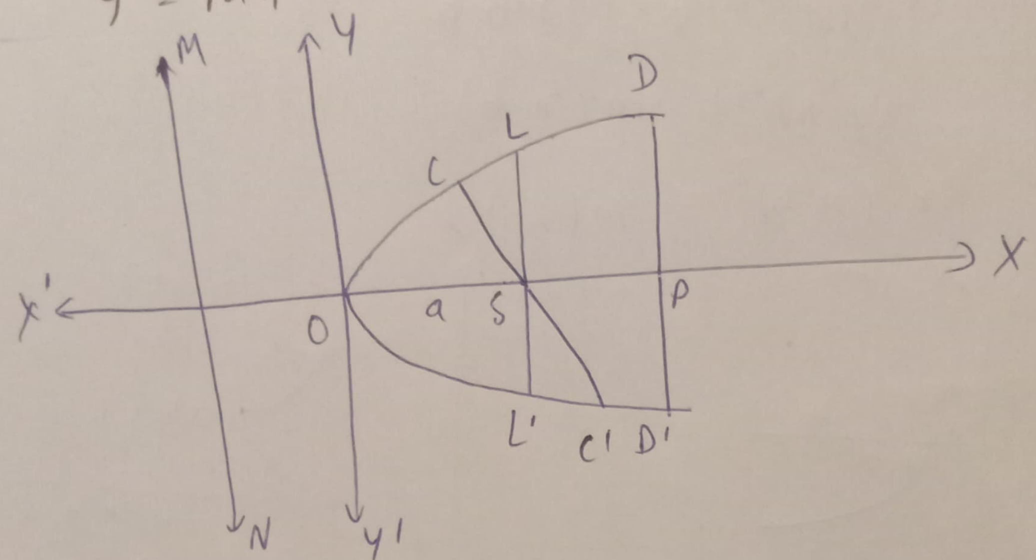


→ A parabola has two real foci situated on its axis, one of which is the focus 'S' & the other lies at infinity; the corresponding directrix is also at infinity.

- Equation of Parabola whose axis is parallel to the x-axis & y-axis is $x = Ay^2 + By + C$ and $y = Ax^2 + Bx + C$

- Terms related to parabola:

For $y^2 = 4ax$



Focus, $S \equiv (a,0)$

Directrix, $MN \equiv x = -a$

Vertex, $O \equiv (0,0)$

Focal chord $\Rightarrow CC'$

Double ordinate, $DD' \rightarrow$ If abscissa of D is h

$y^2 = 4ah \Rightarrow y = \pm (4ah)^{\frac{1}{2}}$

$D \equiv (h, 2\sqrt{ah})$, $D' \equiv (h, -2\sqrt{ah})$

Latus Rectum, $\overline{LL'} = 4a$

$LL' \equiv x = a$

- Intersection of a line & a Parabola ÷

$y^2 = 4ax \quad , \quad y = mx + C$

→ cuts the parabola in two points in real, coincident & imaginary case as

$a >, = , < mc$

→ Condition of tangency : $c = \frac{a}{m}$ $[m \neq 0]$

$y = mx + \frac{a}{m}$ is always a tangent to $y^2 = 4ax$

→ Point of contact : $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

- m-point of parabola

- Equation of tangent in different forms ;

→ Point form ÷ Eq^n of tangent of the parabola $y^2 = 4ax$ at the point (x_1, y_1) is $yy_1 = 2a(x + x_1)$

→ Parametric form ÷ Eq^n of tangent of the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ or t is

$ty = x + at^2$

→ Slope form - Eq^n of tangent of slope m to $y^2 = 4ax$ is $y = mx + \frac{a}{m}$

Point of contact = $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

- Chord of Contact :- The chord of contact of tangents drawn from a point (x_1, y_1) to the parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$.

- length of chord of contact :-

$$X = \frac{\sqrt{(y_1^2 + 4ax_1)(y_1^2 + 4a^2)}}{a}$$

- Equation of Normal :-

→ Point form :- Eqn of normal to the parabola $y^2 = 4ax$ at the point (x_1, y_1) is

$$y - y_1 = -\frac{y_1}{2a}(x - x_1)$$

→ Slope form :- Eqn of normal to the parabola $y^2 = 4ax$ of slope m is $y = mx - 2am - am^3$

& point of contact is $(am^2, -2am)$

- Diameter - Eqⁿ of diameter bisecting chords of slope m of $y^2 = 4ax$ is $y = \frac{2a}{m}$.

- Length of tangent $= y_1 \sqrt{1 + \left(\frac{dx}{dy}\right)^2_{x_1,y_1}}$

  normal $= y_1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2_{x_1,y_1}}$

  Subtangent $= y_1 \left(\frac{dx}{dy}\right)_{x_1,y_1}$

  Subnormal $= y_1 \left(\frac{dy}{dx}\right)_{x_1,y_1}$