Evaluate $\int \frac{x}{(x-1)(x^2+4)} dx$.

Answer: $\int \frac{x}{(x-1)(x^2+4)} dx$

Let $\frac{x}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$

or $x = A(x^2+4) + (Bx+C)(x-1)$

Putting $x = 1$, we get $1 = 5$ A.

Putting $x = 0$, we get $0 = 4A - C$.

Putting $x = -1$, we get $-1 = 5A + 2B - 2C$.

Solving these equations, we obtain $A = \frac{1}{5}, B = -\frac{1}{5}$, and $C = \frac{4}{5}$.

Substituting the values of A, B, and C , we obtain

$$\begin{aligned}\frac{x}{(x-1)(x^2+4)} &= \frac{1}{5(x-1)} + \frac{-\frac{1}{5}x + \frac{4}{5}}{x^2+4} \\ &= \frac{1}{5(x-1)} - \frac{1}{5}\frac{(x-4)}{(x^2+4)}\end{aligned}$$

or $I = \frac{1}{5}\int \frac{1}{x-1} dx - \frac{1}{5}\int \frac{x-4}{x^2+4} dx$

$$\begin{aligned}&= \frac{1}{5}\int \frac{1}{x-1} dx - \frac{1}{10}\int \frac{2x}{x^2+4} dx + \frac{4}{5}\int \frac{1}{x^2+4} dx \\ &= \frac{1}{5}\log|x-1| - \frac{1}{10}\log(x^2+4) + \frac{4}{5} \times \frac{1}{2}\tan^{-1}\frac{x}{2} + C \\ &= \frac{1}{5}\log|x-1| - \frac{1}{10}\log(x^2+4) + \frac{2}{5}\tan^{-1}\left(\frac{x}{2}\right) + C\end{aligned}$$