Integrate $\int \frac{x^3+3x+2}{(x^2+1)^2(x+1)}dx$. [1999 - 5 Marks]

Answer: $I = \int \frac{x^3+3x+2}{(x^2+1)^2(x+1)}dx$.

$$\frac{x^3 + 3x + 2}{(x^2 + 1)^2 (x + 1)} = \frac{A}{x + 1} + \frac{Rx + C}{x^2 + 1} + \frac{Dx + F}{(x^2 + 1)^2}$$

On comparing the coefficient of A, B, C, D and E , we get

$$A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}, D = 0 \text{ and } E = 2$$

$$\therefore \quad I = -\frac{1}{2}\int \frac{1}{x+1}dx + \frac{1}{2}\int \frac{x+1}{x^2+1}dx + 2\int \frac{dx}{(x^2+1)^2}$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log(x^2+1) + \frac{1}{2}\tan^{-1}x + 2I_1 + c_1$$

where $I_1 = \int \frac{dx}{(x^2+1)^2}$,

Now put $x = \tan\theta$

$$\therefore \quad I_1 = \int \frac{\sec^2\theta}{\sec^4\theta}d\theta = \int (\cos^2\theta)\,d\theta = \frac{1}{2}\int (1+\cos 2\theta)d\theta$$

$$\Rightarrow \quad I_1 = \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) = \frac{1}{2}\tan^{-1}x + \frac{1}{4}\cdot\frac{2x}{1+x^2}$$

$$\therefore \quad I = -\frac{1}{2}\log|x+1| + \frac{1}{4}\log(x^2+1) + \frac{3}{2}\tan^{-1}x$$

$$+ \frac{x}{1+x^2} + c \text{ where } c \text{ is constant of integration.}$$

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