If m is a non-zero number and $\int \frac{x^{5m-1}+2x^{4m-1}}{(x^{2m}+x^m+1)^3} dx = f(x) + c$, then $f(x)$ is:

[Main Online April 19, 2014]

(a) $\frac{x^{5m}}{2m(x^{2m}+x^m+1)^2}$

(b) $\frac{x^{4m}}{2m(x^{2m}+x^m+1)^2}$

(c) $\frac{2m(x^{5m}+x^{4m})}{(x^{2m}+x^m+1)^2}$

(d) $\frac{(x^{5m}-x^{4m})}{2m(x^{2m}+x^m+1)^2}$

Answer:

$$\int \frac{x^{5m-1}+2x^{4m-1}}{(x^{2m}+x^m+1)^3} dx = \int \frac{x^{5m-1}+2x^{4m-1}}{x^{6m}(1+x^{-m}+x^{-2m})^3} dx$$

$$= \int \frac{x^{-m-1}+2x^{-2m-1}}{(1+x^{-m}+x^{-2m})^3} dx$$

Put $t = 1 + x^{-m} + x^{-2m}$

$$\therefore \frac{dt}{dx} = -mx^{-m-1} - 2mx^{-2m-1}$$

$$\Rightarrow \frac{dt}{-m} = (x^{-m-1} + 2x^{-2m-1})dx$$

$$\therefore \int \frac{x^{5m-1}+2x^{4m-1}}{(x^{2m}+x^m+1)^3} dx = \frac{1}{-m}\int t^{-3} dt = \frac{1}{2mt^2} +$$

$$= \frac{1}{2m(1+x^{-m}+x^{-2m})^2} + C$$

$$= \frac{x^{4m}}{2m(x^{2m}+x^m+1)^2} + C$$

$$\therefore f(x) = \frac{x^{4m}}{2m(x^{2m}+x^m+1)^2}$$

Option B is the answer