

$$\int \frac{(x+1)}{x(1+xe^x)^2} dx \text{ [1996-2 Marks]}$$

$$\text{Answer: } I = \int \frac{(x+1)}{x(1+xe^x)^2} dx = \int \frac{e^x(x+1)}{xe^x(1+xe^x)^2} dx$$

$$\text{Put } 1 + xe^x = t \Rightarrow (xe^x + e^x)dx = dt$$

$$\therefore I = \int \frac{dt}{(t-1)t^2} = \int \left(\frac{1}{1-t} + \frac{1}{t} + \frac{1}{t^2} \right) dt$$

$$= -\log|1-t| + \log|t| - \frac{1}{t} + c$$

$$= \log\left|\frac{t}{1-t}\right| - \frac{1}{t} + c = \log\left|\frac{1+xe^x}{-xe^x}\right| - \frac{1}{1+xe^x} + c$$

$$= \log\left(\frac{1+xe^x}{xe^x}\right) - \frac{1}{1+xe^x} + c$$