

11. A uniformly tapering conical wire is made from a material of Young's modulus Y and has a normal, unextended length L . The radii, at the upper and lower ends of this conical wire, have values R and $3R$, respectively. The upper end of the wire is fixed to a rigid support and a mass M is suspended from its lower end. The equilibrium extended length, of this wire, would equal : **[Online April 9, 2016]**

(a) $L \left(1 + \frac{2}{9} \frac{Mg}{\pi Y R^2} \right)$

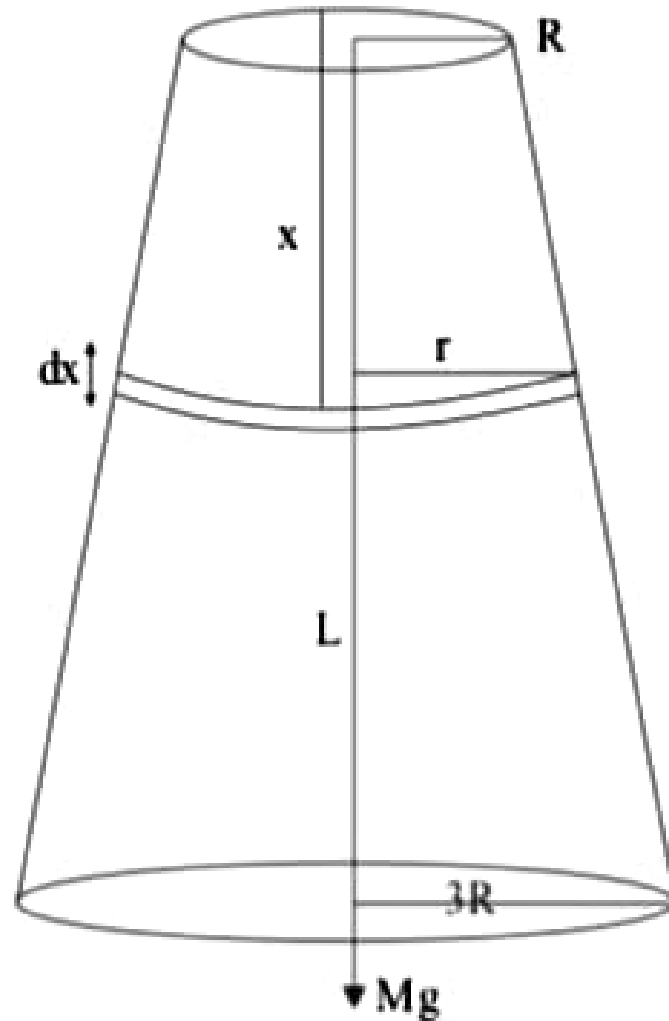
(b) $L \left(1 + \frac{1}{9} \frac{Mg}{\pi Y R^2} \right)$

(c) $L \left(1 + \frac{1}{3} \frac{Mg}{\pi Y R^2} \right)$

(d) $L \left(1 + \frac{2}{3} \frac{Mg}{\pi Y R^2} \right)$

11. (c) Consider a small element dx of radius r ,

$$r = \frac{2R}{L}x + R$$



At equilibrium change in length of the wire

$$\int_0^l dL = \int \frac{Mg dx}{\pi \left[\frac{2R}{L} x + R \right]^2 y}$$

Taking limit from 0 to L

$$\Delta L = \frac{Mg}{\pi y} \left[-\frac{1}{\left[\frac{2Rx}{L} + R \right]^2} \times \frac{L}{2R} \right]_0^L = \frac{MgL}{3\pi R^2 y}$$

The equilibrium extended length of wire = $L + \Delta L$

$$= L + \frac{MgL}{3\pi R^2 Y} = L \left(1 + \frac{1}{3} \frac{Mg}{\pi Y R^2} \right)$$