The coefficient of x^7 in the expansion of $(1 - ax)^{-7}$ is $\frac{13!}{7! \, 6!} 2^7$ then find the value of a (|ax| < 1)

SOLUTION :

we know that coefficient of t^r in the expansion (1-t)⁻ⁿ is $n+r-1_{\mathcal{C}_r}$ therefore

Coefficient of $(ax)^7$ in the expansion $(1-ax)^{-7}$ is $7 + 7 - 1_{C_7} = 13_{C_7}$ therefore

$$a^7 \ 13_{C_7} = \frac{13!}{7! \ 6!} 2^7$$

 $a^7 = 2^7$

a=2