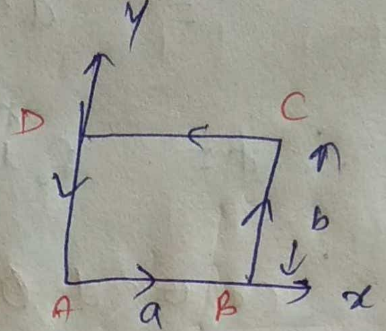
Q. Is it is possible to have an electrostatic field of the form $\vec{E} = E_0 x \hat{j}$.

→ Electrostatic field satisfy

$$\boxed{\oint \vec{E}\cdot d\vec{l} = 0}$$

Let us ~~condition~~ consider a ractangular path.



$$\oint \vec{E}\cdot d\vec{l} = \int_A^B \vec{E}\cdot d\vec{l} + \int_B^C \vec{E}\cdot d\vec{l} + \int_C^D \vec{E}\cdot d\vec{l} + \int_D^A \vec{E}\cdot d\vec{l}$$

$$\int_A^B \vec{E}\cdot d\vec{l} = \int E_0 x \hat{j}\cdot\hat{i}\, dx = 0$$

similarily

$$\int_C^D \vec{E}\cdot d\vec{l} = 0$$

$$\int_B^C \vec{E}\cdot d\vec{l} = \int_0^b E_0 a \hat{j}\cdot\hat{j}\, dy$$

$$= E_0 ab$$

$$\int_D^A \vec{E}\cdot d\vec{l} = 0$$

$$\Rightarrow \boxed{\oint \vec{E}\cdot d\vec{l} = E_0 ab \neq 0}$$

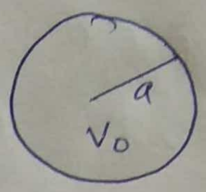
So E is not represent an electrostatic field,

Q. Given the following electrostatic potential, calculate the corresponding electric field:—

$$V = V_0 \quad \text{for } r = \sqrt{x^2+y^2+z^2} < a$$

$$= \frac{V_0 a}{\sqrt{x^2+y^2+z^2}} \;; \quad r = \sqrt{x^2+y^2+z^2} > a$$

As
$$E_x = -\frac{\partial V}{\partial x}$$
$$E_y = -\frac{\partial V}{\partial y}$$
$$E_z = -\frac{\partial V}{\partial z}$$



$V(r)$

a) For $r < a$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial (V_0)}{\partial x} = 0$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial V_0}{\partial y} = 0$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial V_0}{\partial z} = 0$$

$$\Rightarrow \boxed{E = 0 \quad \text{for } r < a}$$

b) For $r > a$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left[\frac{V_0 a}{\sqrt{x^2+y^2+z^2}}\right]$$

$$= \frac{V_0 a x}{(x^2+y^2+z^2)^{3/2}} = \frac{V_0 a x}{r^3}$$

similarily:
$$E_y = \frac{V_0 a y}{r^3}, \quad E_z = \frac{V_0 a z}{r^3}$$

$$\therefore \vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

$$\boxed{\vec{E} = \frac{V_0 a}{r^3}\vec{r}}$$

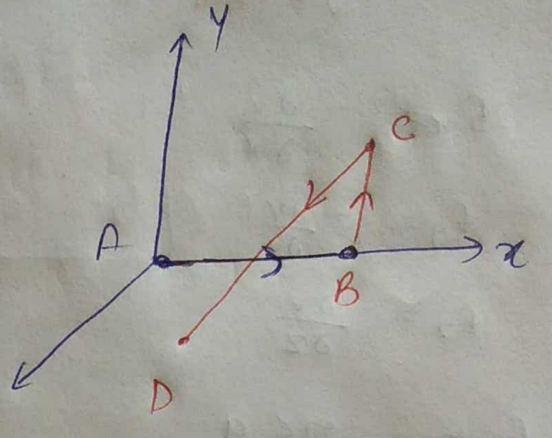
$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Ans

Q. Consider an electrostatic field given by

$$\vec{E} = (20\hat{i} + 30\hat{j})\ V/m$$

Calculate the potential difference b/w the origin and the point P with coordinate $x = 2m$, $y = 2m$, $Z = 2m$.



$$V = -\int_A^D \vec{E}\cdot d\vec{l}$$

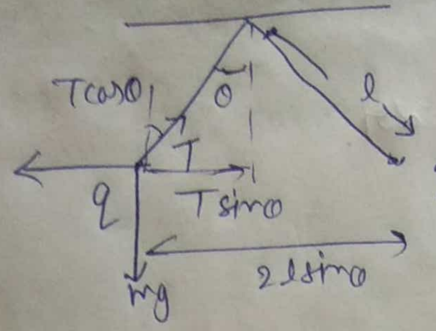
$$= -\int_A^B \vec{E}\cdot\hat{i}\,dx - \int_B^C \vec{E}\cdot\hat{j}\,dy - \int_C^D \vec{E}\cdot\hat{k}\,dz$$

$$= -\int_0^x 20\,dx - \int_0^2 30\,dy - 0$$

$$= -40 - 60$$

$$\boxed{V = -100 \text{ Volt}}$$ Ans

Q. Two point charges of equal mass m and carrying equal charges q are suspended from a common point by two strings having negligible mass and of Length l. obtain an expression relating q and θ at equilibrium.



$$T\cos\theta = mg \quad , \quad T\sin\theta = F_e = \frac{q^2}{4\pi\varepsilon_0 (2l\sin\theta)^2}$$

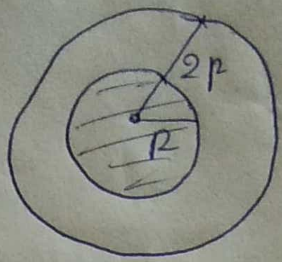
$$= \frac{q^2}{16\pi\varepsilon_0 l^2 \sin^2\theta}$$

$$\therefore \tan\theta = \frac{T\sin\theta}{T\cos\theta}$$

$$\tan\theta = \frac{q^2}{16\pi\varepsilon_0 l^2 \sin^2\theta} \cdot \frac{1}{mg}$$

$$\boxed{q^2 = 16\pi\varepsilon_0 l^2 mg \sin^2\theta \cdot \tan\theta}$$

Q. three charge is embedded in a linear dielectric sphere of dielectric constant K_1 and radius R, The free charge density $\rho_f = \alpha r$, where $\alpha =$ constant and r is the distance from the centre. This sphere is surrounded by another spherical shell of radii R and $2R$ and of dielectric constant K_2. Calculate the electric field $\vec{E}$ and the displacement vector $\vec{D}$ everywhere.



$\rho_f = \alpha r$

As we know gauss law

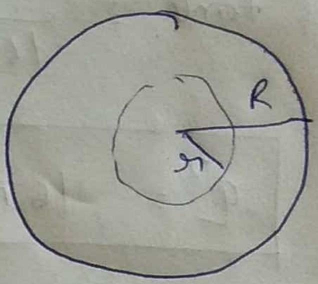
$$\oint \vec{D}\cdot d\vec{A} = Q_{f\,enclosed}$$

And $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$\vec{D}$, $\vec{E}$ & $\vec{P}$ will be along radial direction

& will only depends on r..

a) <u>for $0 < r < R$</u>



$$\oint \vec{D}\cdot d\vec{A} = Q_{f\,enclosed}$$

$$D\cdot 4\pi r^2 = \int_0^r \alpha r \cdot 4\pi r^2 dr$$

$$= 4\pi\alpha \int_0^r r^3 dr$$

$$4\pi r^2 \cdot D = \pi\alpha r^4$$

$$\cancel{\vec{D} = \frac{\alpha}{4} r} \quad D = \frac{\alpha}{4} r^2$$

$$\boxed{\vec{D} = \frac{\alpha}{4} r^2 \hat{r}}$$

Now $\boxed{\vec{E} = \frac{\vec{D}}{\varepsilon_0 K_1} = \frac{\alpha r^2}{4\varepsilon_0 K_1}\hat{r}}$

$$\vec{P} = \vec{D} - \varepsilon_0\vec{E} = \varepsilon_0 (K_1 - 1)\vec{E}$$

$$\boxed{\vec{P} = \frac{(K_1 - 1)}{4K_1}\alpha r^2 \hat{r}}$$

polarisation.

b) $R < r < 2R$:—

$$\oint \vec{D}\cdot d\vec{A} = Q_{enclosed}$$

$$4\pi r^2 D = \int_0^R \alpha r \cdot 4\pi r^2 \, dr$$

$$\boxed{\vec{D} = \frac{\alpha}{4}\frac{R^4}{r^2}\hat{r}}$$

$$\boxed{\vec{E} = \frac{\vec{D}}{\varepsilon_0 K_2} = \frac{\alpha R^4}{4\varepsilon_0 K_2 r^2}\hat{r}}$$

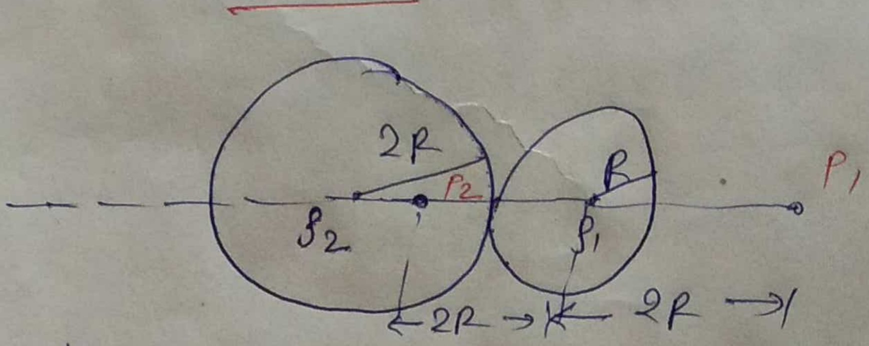
c) $r > 2R$

$$\oint \vec{D}\cdot d\vec{A} = Q_{f\ enclosed}$$

$$4\pi r^2 . D = \pi \alpha R^4$$

$$\boxed{\vec{D} = \frac{\alpha}{4}\frac{R^4}{r^2}\hat{r}}$$

Q. Two non-conducting solid sphere of radii R & $2R$ having uniform volume charge densities ρ_1 & ρ_2 resp. touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere along the line joining the centre is zero. obtain ρ_1/ρ_2.



According to the question, Electric field is zero at point P_1 & P_2. (figure)

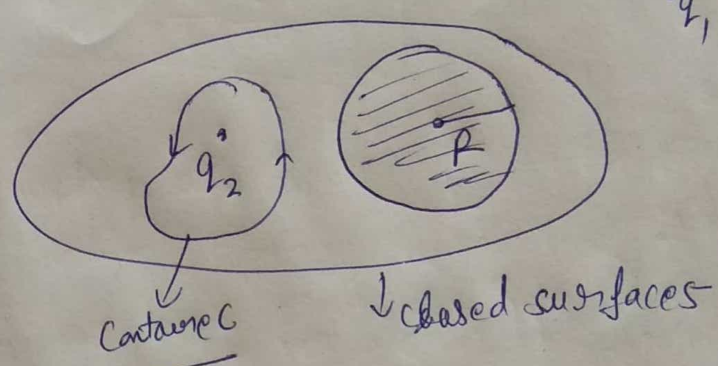
Q. Consider a set of two point charges q_1 & q_2 and a uniformly charged sphere of radius R with uniform volume charge density ρ. Obtain the values of

a) $\oint \vec{E}\cdot d\vec{A}$ over the closed surface S.

b) $\oint \vec{E}\cdot d\vec{l}$ over curve C.

c) For what value of ρ will $\oint \vec{E}\cdot d\vec{A}$ over S vanish?

d) Draw a closed surface through which $\oint \vec{E}\cdot d\vec{A}$ will be independent of ρ.



a) Now $\oint_S \vec{E}\cdot d\vec{A} = \dfrac{Q}{\varepsilon_0} = \dfrac{1}{\varepsilon_0}\left(q_2 + \dfrac{4}{3}\pi R^3 \rho\right)$

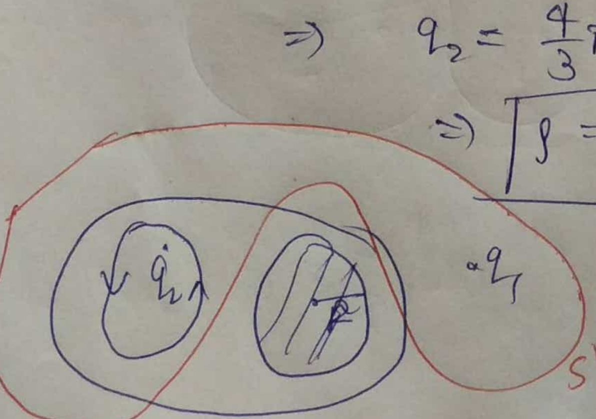
b) $\oint \vec{E}\cdot d\vec{l} = 0$ (for electrostatic field)

c) $\oint \vec{E}\cdot d\vec{A} = 0 \Rightarrow \dfrac{1}{\varepsilon_0}\left(q_2 + \dfrac{4}{3}\pi R^3 \rho\right) = 0$

$\Rightarrow q_2 = \dfrac{4}{3}\pi R^3 \rho$

$\Rightarrow \boxed{\rho = \dfrac{3q_2}{4\pi R^3}}$

d)



$\oint_{S'} \vec{E}\cdot d\vec{A} = \dfrac{1}{\varepsilon_0}(q_1+q_2)$

independent of ρ.