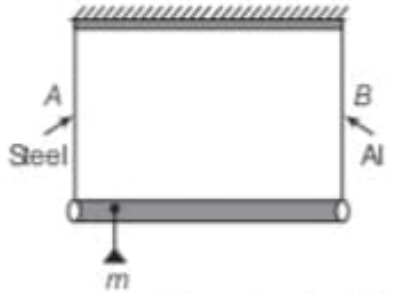
Q. 11 A rod of length l and negligible mass is suspended at its two ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths (figure). The cross-sectional areas of wires A and B are 1.0 mm^2 and 2.0 mm^2, respectively. ($Y_{Al} = 70 \times 10^9 Nm^{-2}$ and $Y_{steel} = 200 \times 10^9 Nm^{-2}$)



(a) Mass m should be suspended close to wire A to have equal stresses in both the wires

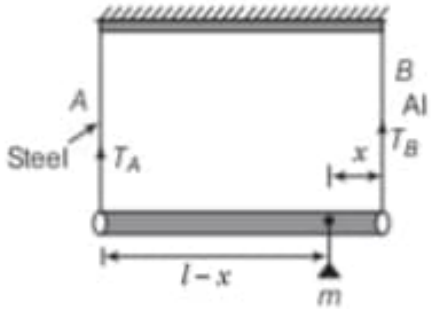
(b) Mass m should be suspended close to B to have equal stresses in both the wires

(c) Mass m should be suspended at the middle of the wires to have equal stresses in both the wires

(d) Mass m should be suspended close to wire A to have equal strain in both wires

Ans. *(b, d)*

Let the mass is placed at x from the end B.



Let T_A and T_B be the tensions in wire A and wire B respectively.

For the rotational equilibrium of the system,

$$\Sigma \tau = 0 \qquad \text{(Total torque = 0)}$$

$$\Rightarrow \quad T_B x - T_A (l - x) = 0$$

$$\Rightarrow \quad \frac{T_B}{T_A} = \frac{l - x}{x} \qquad \ldots(i)$$

$$\text{Stress in wire } A = S_A = \frac{T_A}{a_A}$$

$$\text{Stress in wire } B = S_B = \frac{T_B}{a_B}$$

where a_A and a_B are cross-sectional areas of wire A and B respectively.

By question $a_B = 2\, a_A$

Now, **for equal stress** $S_A = S_B$

$$\Rightarrow \quad \frac{T_A}{a_A} = \frac{T_B}{a_B} \Rightarrow \frac{T_B}{T_A} = \frac{a_B}{a_A} = 2$$

$$\Rightarrow \quad \frac{l - x}{x} = 2 \Rightarrow \frac{l}{x} - 1 = 2$$

$$\Rightarrow \quad x = \frac{l}{3} \Rightarrow l - x = l - l/3 = \frac{2l}{3}$$

Hence, mass m should be placed closer to B.

For equal strain, $(\text{strain})_A = (\text{strain})_B$

$$\Rightarrow \quad \frac{(Y_A)}{S_A} = \frac{Y_B}{S_B} \qquad \text{(where } Y_A \text{ and } Y_B \text{ are Young modulii)}$$

$$\Rightarrow \quad \frac{Y_{\text{steel}}}{T_A/a_A} = \frac{Y_{\text{Al}}}{T_B/a_B}$$

$$\Rightarrow \quad \frac{Y_{\text{steel}}}{Y_{\text{Al}}} = \frac{T_A}{T_B} \times \frac{a_B}{a_A} = \left(\frac{x}{l-x}\right)\left(\frac{2a_A}{a_A}\right)$$

$$\Rightarrow \quad \frac{200 \times 10^9}{70 \times 10^9} = \frac{2x}{l-x} \Rightarrow \frac{20}{7} = \frac{2x}{l-x}$$

$$\Rightarrow \quad \frac{10}{7} = \frac{x}{l-x} \Rightarrow 10l - 10x = 7x$$

$$\Rightarrow \quad 17x = 10l \Rightarrow x = \frac{10\,l}{17}$$

$$l - x = l - \frac{10l}{17} = \frac{7l}{17}$$

Hence, mass m should be placed closer to wire A.