Concepts and Formulas to Remember :

Binomial Distribution Formula	
Binomial Distribution	$P(x) = {}^nC_x \cdot p^x (1 - p)^{n-x}$
Or,	$P(r) = [n!/r!(n-r)!] \cdot p^r (1 - p)^{n-r}$

Where,

- n = Total number of events
- r (or) x = Total number of successful events.
- p = Probability of success on a single trial.
- $^nC_r = [n!/r!(n-r)]!$
- 1 – p = Probability of failure.

➔ **This concept can be best cleared by some examples, some are given below using the above formulas :**

Example 1:

A coin is tossed12 times. What is the probability of getting exactly 7 heads?

Solution:

Given that a coin is tossed 12 times. (i.e) n= 12

Thus, a probability pf gettig head in single toss = ½. (i.e) p = ½.

So, 1-p = 1-½ = ½.

We know that the binomial probability distribution is $P(r) = {}^nC^r \cdot p^r (1 - p)^{n-r}$.

Now, we have to find the probability of getting exactly 7 heads.(i.e) r = 7.

Substituting the values in the binomial distribution formula, we get

$P(7) = {}^{12}C_7 \cdot (½)^7 (½)^{12-7}$

$P(7) = 792 \cdot (½)^7 (½)^5$

$P(7) = 792.(½)^{12}$

P(7) = 792 (1/4096)

P(7) = 0.193

Therefore, the probability of getting exactly 7 heads is 0.193.

Example 2:

A coin that is fair in nature is tossed n number of times. The probability of the occurrence of a head six times is the same as the probability that a head comes 8 times, then find the value of n.

Solution:

The probability that head occurs 6 times = $^nC_6 (½)^6 (½)^{n-6}$

Similarly, the probability that head occurs 8 times = $^nC_8 (½)^8 (½)^{n-8}$

Given that, the probability of the occurrence of a head six times is the same as the probability that a head comes 8 times,

(i.e) $^nC_6 (½)^6 (½)^{n-6} = {}^nC_8 (½)^8 (½)^{n-8}$

$\Rightarrow {}^nC_6(½)^n = {}^nC_8 (½)^n$

$\Rightarrow {}^nC_6 = {}^nC_8$

$\Rightarrow 6 = n-8$

$\Rightarrow n= 14.$

Therefore, the value of n is 14.

Example 3:

The probability that a person can achieve a target is 3/4. The count of tries is 5. What is the probability that he will attain the target at least thrice?

Solution:

Given that, p = ¾, q = ¼, n = 5.

Using binomial distribution formula, we get $P(X) = {}^nC_x \cdot p^x (1 - p)^{n-x}$

Thus, the required probability is: P(X = 3) + P(X=4) + P(X=5)

$= {}^5C_3 \cdot (¾)^3 (¼)^2 + {}^5C_4 \cdot (¾)^4 (¼)^1 + {}^5C_5 \cdot (¾)^5 = 459/512$

Probability Distributions: The system in which the values of a random variable are given along with their corresponding probabilities is called probability distribution.
Let X be a random variable which can take n values $x_1, x_2, \ldots, x_n$.
Let $p_1, p_2, \ldots, p_n$ be the respective probabilities.
Then, a probability distribution table is given as follows:

X	x_1	x_2	x_3	...	x_n
P	p_1	p_2	p_3	...	p_n

such that $P_1 + p_2 + P_3 + \ldots + p_n = 1$
Note: If x_i is one of the possible values of a random variable X, then statement $X = x_i$ is true only at some point(s) of the sample space. Hence ,the probability that X takes value x, is always non-zero, i.e. $P(X = x_i) \neq 0$

Mean and Variance of a Probability Distribution: Mean of a probability distribution is

$\sum_{i=1}^{n} x_i \cdot p_i$. It is also called expectation of X, i.e. $E(X) = \mu = \sum_{i=1}^{n} x_i p_i$.

Variance is given by $V(X)$ or σ_x^2 and defined as $\sigma^2 = \sum_{i=1}^{n} x_i^2 \cdot p(x_i) - \left(\sum_{i=1}^{n} x_i \cdot p(x_i)\right)^2$

or $$E(X^2) - [E(X)]^2$$

or Equivalently $V(X) = E(X-\mu)^2$ or $\sum_{i=1}^{n} (x_i - \mu)^2 \, p(x_i)$

Standard deviation is given by, $\sigma_x = \sqrt{v(x)} = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 \, p(x_i)}$

Bernoulli Trial: Trials of a random experiment are called Bernoulli trials if they satisfy the following conditions:
(i) There should be a finite number of trials.
(ii) The trials should be independent.
(iii) Each trial has exactly two outcomes, success or failure.
(iv) The probability of success remains the same in each trial.

Binomial Distribution: The probability distribution of numbers of successes in an experiment consisting of n Bernoulli trials obtained by the binomial expansion (p + q)n, is called binomial distribution.
Let X be a random variable which can take n values $x_1, x_2, \ldots, x_n$. Then, by binomial distribution, we have $P(X = r) = {}^nC_r \, p^r q^{n-r}$
where,

n = Total number of trials in an experiment
p = Probability of success in one trial
q = Probability of failure in one trial
r = Number of success trial in an experiment
Also, p + q = 1
Binomial distribution of the number of successes X can be represented as

X	0	1	2	... r...	n
$P(X)$	${}^nC_0p^0q^n$	${}^nC_1p^1q^{n-1}$	${}^nC_2p^2q^{n-2}$	... ${}^nC_rp^rq^{n-r}$ ...	${}^nC_np^n$

Mean and Variance of Binomial Distribution
(i) Mean(μ) = $\Sigma x_i p_i = np$
(ii) Variance(σ^2) = $\Sigma x_i^2 p_i - \mu^2 = npq$
(iii) Standard deviation (σ) = $\sqrt{\text{Variance}} = \sqrt{npq}$
Note: Mean > Variance