12090CH12

Chapter Twelve

ATOMS

12.1 Introduction

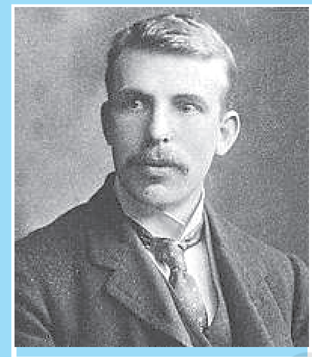
By the nineteenth century, enough evidence had accumulated in favour of atomic hypothesis of matter. In 1897, the experiments on electric discharge through gases carried out by the English physicist J. J. Thomson (1856 – 1940) revealed that atoms of different elements contain negatively charged constituents (electrons) that are identical for all atoms. However, atoms on a whole are electrically neutral. Therefore, an atom must also contain some positive charge to neutralise the negative charge of the electrons. But what is the arrangement of the positive charge and the electrons inside the atom? In other words, what is the structure of an atom?

The first model of atom was proposed by J. J. Thomson in 1898. According to this model, the positive charge of the atom is uniformly distributed throughout the volume of the atom and the negatively charged electrons are embedded in it like seeds in a watermelon. This model was picturesquely called *plum pudding model* of the atom. However subsequent studies on atoms, as described in this chapter, showed that the distribution of the electrons and positive charges are very different from that proposed in this model.

We know that condensed matter (solids and liquids) and dense gases at all temperatures emit electromagnetic radiation in which a continuous distribution of several wavelengths is present, though with different intensities. This radiation is considered to be due to oscillations of atoms and molecules, governed by the interaction of each atom or molecule with its neighbours. *In contrast*, light emitted from rarefied gases heated in a flame, or excited electrically in a glow tube such as the familiar neon sign or mercury vapour light has only certain discrete wavelengths. The spectrum appears as a series of bright lines. In such gases, the average spacing between atoms is large. Hence, the radiation emitted can be considered due to individual atoms rather than because of interactions between atoms or molecules.

In the early nineteenth century it was also established that each element is associated with a characteristic spectrum of radiation, for example, hydrogen always gives a set of lines with fixed relative position between the lines. This fact suggested an intimate relationship between the internal structure of an atom and the spectrum of radiation emitted by it. In 1885, Johann Jakob Balmer (1825 – 1898) obtained a simple empirical formula which gave the wavelengths of a group of lines emitted by atomic hydrogen. Since hydrogen is simplest of the elements known, we shall consider its spectrum in detail in this chapter.

Ernst Rutherford (1871–1937), a former research student of J. J. Thomson, was engaged in experiments on α-particles emitted by some radioactive elements. In 1906, he proposed a classic experiment of scattering of these α-particles by atoms to investigate the atomic structure. This experiment was later performed around 1911 by Hans Geiger (1882–1945) and Ernst Marsden (1889–1970, who was 20 year-old student and had not yet earned his bachelor's degree). The details are discussed in Section 12.2. The explanation of the results led to the birth of Rutherford's planetary model of atom (also called the *nuclear model of the atom*). According to this the entire positive charge and most of the mass of the atom is concentrated in a small volume called the nucleus with electrons revolving around the nucleus just as planets revolve around the sun.



Ernst Rutherford (1871 – 1937) New Zealand born, British physicist who did pioneering work on radioactive radiation. He discovered alpha-rays and beta-rays. Along with Federick Soddy, he created the modern theory of radioactivity. He studied the 'emanation' of thorium and discovered a new noble gas, an isotope of radon, now known as thoron. By scattering alpha-rays from the metal foils, he discovered the atomic nucleus and proposed the plenatery model of the atom. He also estimated the approximate size of the nucleus.

ERNST RUTHERFORD (1871 – 1937)

Rutherford's nuclear model was a major step towards how we see the atom today. However, it could not explain why atoms emit light of only discrete wavelengths. How could an atom as simple as hydrogen, consisting of a single electron and a single proton, emit a complex spectrum of specific wavelengths? In the classical picture of an atom, the electron revolves round the nucleus much like the way a planet revolves round the sun. However, we shall see that there are some serious difficulties in accepting such a model.

12.2 Alpha-particle Scattering and Rutherford's Nuclear Model of Atom

At the suggestion of Ernst Rutherford, in 1911, H. Geiger and E. Marsden performed some experiments. In one of their experiments, as shown in

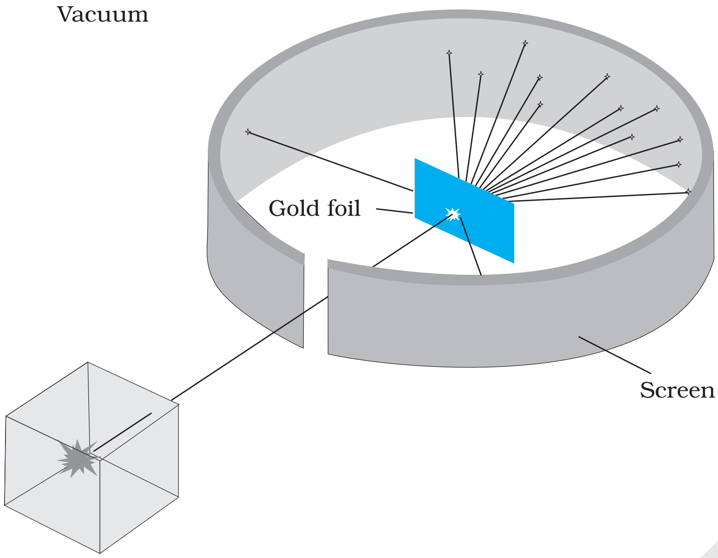


FIGURE 12.1 Geiger-Marsden scattering experiment. The entire apparatus is placed in a vacuum chamber (not shown in this figure).

Fig. 12.1, they directed a beam of 5.5 MeV α-particles emitted from a $^{214}_{83}$Bi radioactive source at a thin metal foil made of gold. Figure 12.2 shows a schematic diagram of this experiment. Alpha-particles emitted by a $^{214}_{83}$Bi radioactive source were collimated into a narrow beam by their passage through lead bricks. The beam was allowed to fall on a thin foil of gold of thickness 2.1×10^{-7} m. The scattered alpha-particles were observed through a rotatable detector consisting of zinc sulphide screen and a microscope. The scattered alpha-particles on striking the screen produced brief light flashes or scintillations. These flashes may be viewed through a microscope and the distribution of the number of scattered particles may be studied as a function of angle of scattering.

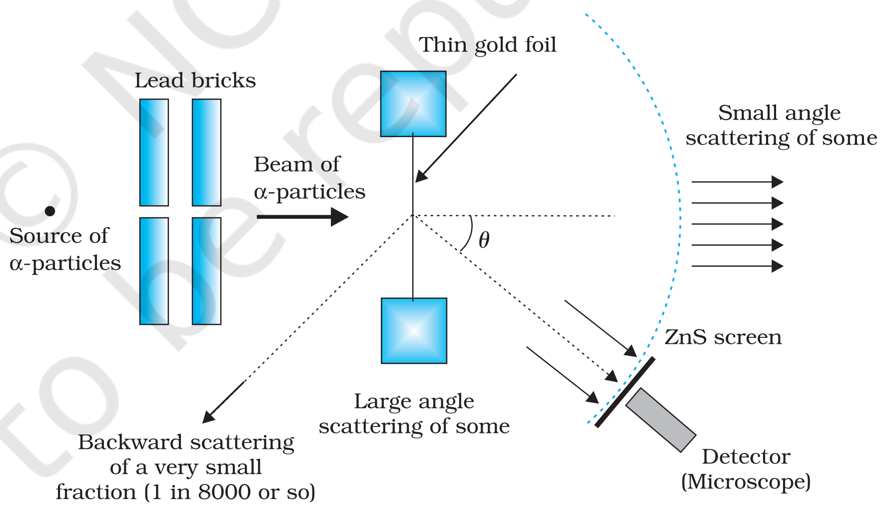


FIGURE 12.2 Schematic arrangement of the Geiger-Marsden experiment.

A typical graph of the total number of α-particles scattered at different angles, in a given interval of time, is shown in Fig. 12.3. The dots in this figure represent the data points and the solid curve is the theoretical prediction based on the assumption that the target atom has a small, dense, positively charged nucleus. Many of the α-particles pass through the foil. It means that they do not suffer any collisions. Only about 0.14% of the incident α-particles scatter by more than 1°; and about 1 in 8000 deflect by more than 90°. Rutherford argued that, to deflect the α-particle backwards, it must experience a large repulsive force. This force could

2022-23

be provided if the greater part of the mass of the atom and its positive charge were concentrated tightly at its centre. Then the incoming α-particle could get very close to the positive charge without penetrating it, and such a close encounter would result in a large deflection. This agreement supported the hypothesis of the nuclear atom. This is why Rutherford is credited with the *discovery* of the nucleus.

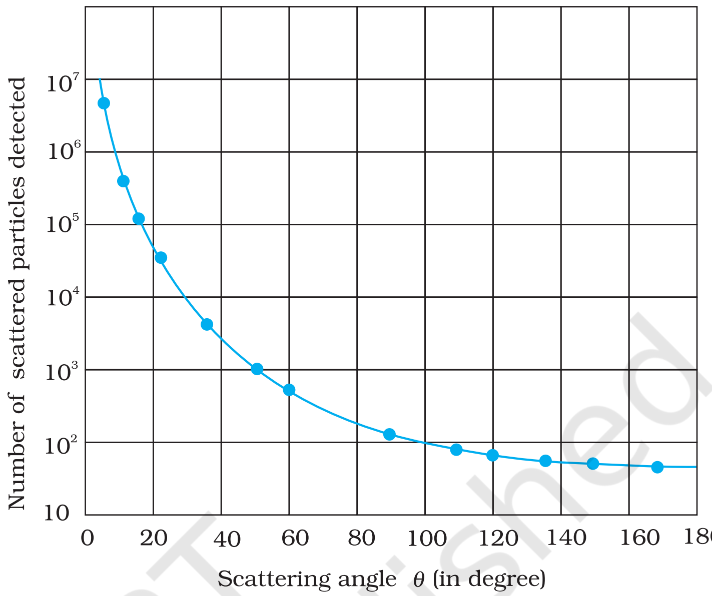


FIGURE 12.3 Experimental data points (shown by dots) on scattering of α-particles by a thin foil at different angles obtained by Geiger and Marsden using the setup shown in Figs. 12.1 and 12.2. Rutherford's nuclear model predicts the solid curve which is seen to be in good agreement with experiment.

In Rutherford's nuclear model of the atom, the entire positive charge and most of the mass of the atom are concentrated in the nucleus with the electrons some distance away. The electrons would be moving in orbits about the nucleus just as the planets do around the sun. Rutherford's experiments suggested the size of the nucleus to be about 10^{-15} m to 10^{-14} m. From kinetic theory, the size of an atom was known to be 10^{-10} m, about 10,000 to 100,000 times larger than the size of the nucleus (see Chapter 11, Section 11.6 in Class XI Physics textbook). Thus, the electrons would seem to be at a distance from the nucleus of about 10,000 to 100,000 times the size of the nucleus itself. Thus, most of an atom is empty space. With the atom being largely empty space, it is easy to see why most α-particles go right through a thin metal foil. However, when α-particle happens to come near a nucleus, the intense electric field there scatters it through a large angle. The atomic electrons, being so light, do not appreciably affect the α-particles.

The scattering data shown in Fig. 12.3 can be analysed by employing Rutherford's nuclear model of the atom. As the gold foil is very thin, it can be assumed that α-particles will suffer not more than one scattering during their passage through it. Therefore, computation of the trajectory of an alpha-particle scattered by a single nucleus is enough. Alpha-particles are nuclei of helium atoms and, therefore, carry two units, $2e$, of positive charge and have the mass of the helium atom. The charge of the gold nucleus is Ze, where Z is the atomic number of the atom; for gold $Z = 79$. Since the nucleus of gold is about 50 times heavier than an α-particle, it is reasonable to assume that it remains stationary throughout the scattering process. Under these assumptions, the trajectory of an alpha-particle can be computed employing Newton's second law of motion and the Coulomb's law for electrostatic force of repulsion between the alpha-particle and the positively charged nucleus.

The magnitude of this force is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(Ze)}{r^2} \qquad (12.1)$$

where r is the distance between the α-particle and the nucleus. The force is directed along the line joining the α-particle and the nucleus. The magnitude and direction of the force on an α-particle continuously changes as it approaches the nucleus and recedes away from it.

12.2.1 Alpha-particle trajectory

The trajectory traced by an α-particle depends on the impact parameter, *b* of collision. The *impact parameter* is the perpendicular distance of the initial velocity vector of the α-particle from the centre of the nucleus (Fig. 12.4). A given beam of α-particles has a distribution of impact parameters *b*, so that the beam is scattered in various directions with different probabilities (Fig. 12.4). (In a beam, all particles have nearly same kinetic energy.) It is seen that an α-particle close to the nucleus (small impact parameter) suffers large scattering. In case of head-on collision, the impact parameter is minimum and the α-particle rebounds back ($\theta \cong \pi$). For a large impact parameter, the α-particle goes nearly undeviated and has a small deflection ($\theta \cong 0$).

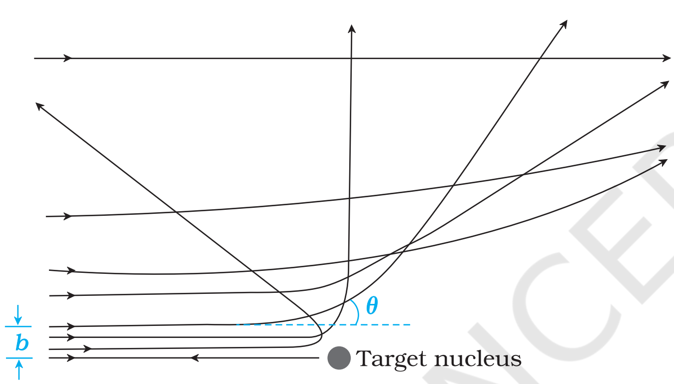


FIGURE 12.4 Trajectory of α-particles in the coulomb field of a target nucleus. The impact parameter, *b* and scattering angle *θ* are also depicted.

The fact that only a small fraction of the number of incident particles rebound back indicates that the number of α-particles undergoing head on collision is small. This, in turn, implies that the mass and positive charge of the atom is concentrated in a small volume. Rutherford scattering therefore, is a powerful way to determine an upper limit to the size of the nucleus.

Example 12.1 In the Rutherford's nuclear model of the atom, the nucleus (radius about 10^{-15} m) is analogous to the sun about which the electron move in orbit (radius ≈ 10^{-10} m) like the earth orbits around the sun. If the dimensions of the solar system had the same proportions as those of the atom, would the earth be closer to or farther away from the sun than actually it is? The radius of earth's orbit is about 1.5×10^{11} m. The radius of sun is taken as 7×10^{8} m.

Solution The ratio of the radius of electron's orbit to the radius of nucleus is $(10^{-10}\text{ m})/(10^{-15}\text{ m}) = 10^5$, that is, the radius of the electron's orbit is 10^5 times larger than the radius of nucleus. If the radius of the earth's orbit around the sun were 10^5 times larger than the radius of the sun, the radius of the earth's orbit would be $10^5 \times 7 \times 10^8$ m = 7×10^{13} m. This is more than 100 times greater than the actual orbital radius of earth. Thus, the earth would be much farther away from the sun.

It implies that an atom contains a much greater fraction of empty space than our solar system does.

EXAMPLE 12.1

Example 12.2 In a Geiger-Marsden experiment, what is the distance of closest approach to the nucleus of a 7.7 MeV α-particle before it comes momentarily to rest and reverses its direction?

Solution The key idea here is that throughout the scattering process, the total mechanical energy of the system consisting of an α-particle and a gold nucleus is conserved. The system's initial mechanical energy is E_i, before the particle and nucleus interact, and it is equal to its mechanical energy E_f when the α-particle momentarily stops. The initial energy E_i is just the kinetic energy K of the incoming α- particle. The final energy E_f is just the electric potential energy U of the system. The potential energy U can be calculated from Eq. (12.1).

Let d be the centre-to-centre distance between the α-particle and the gold nucleus when the α-particle is at its stopping point. Then we can write the conservation of energy $E_i = E_f$ as

$$K = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(Ze)}{d} = \frac{2Ze^2}{4\pi\varepsilon_0 d}$$

Thus the distance of closest approach d is given by

$$d = \frac{2Ze^2}{4\pi\varepsilon_0 K}$$

The maximum kinetic energy found in α-particles of natural origin is 7.7 MeV or 1.2×10^{-12} J. Since $1/4\pi\varepsilon_0 = 9.0 \times 10^9$ N m^2/C^2. Therefore with $e = 1.6 \times 10^{-19}$ C, we have,

$$d = \frac{(2)(9.0\times10^9\,\text{Nm}^2/C^2)(1.6\times10^{-19}\,\text{C})^2 Z}{1.2\times10^{-12}\,\text{J}}$$

$$= 3.84 \times 10^{-16}\, Z\,\text{m}$$

The atomic number of foil material gold is $Z = 79$, so that

d (Au) = 3.0×10^{-14} m = 30 fm. (1 fm (i.e. fermi) = 10^{-15} m.)

The radius of gold nucleus is, therefore, less than 3.0×10^{-14} m. This is not in very good agreement with the observed result as the actual radius of gold nucleus is 6 fm. The cause of discrepancy is that the distance of closest approach is considerably larger than the sum of the radii of the gold nucleus and the α-particle. Thus, the α-particle reverses its motion without ever actually *touching* the gold nucleus.

EXAMPLE 12.2

12.2.2 Electron orbits

The Rutherford nuclear model of the atom which involves classical concepts, pictures the atom as an electrically neutral sphere consisting of a very small, massive and positively charged nucleus at the centre surrounded by the revolving electrons in their respective dynamically stable orbits. The electrostatic force of attraction, F_e between the revolving electrons and the nucleus provides the requisite centripetal force (F_c) to keep them in their orbits. Thus, for a dynamically stable orbit in a hydrogen atom

$$F_e = F_c$$

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r} \qquad (12.2)$$

Thus the relation between the orbit radius and the electron velocity is

$$r = \frac{e^2}{4\pi\varepsilon_0 m v^2} \qquad (12.3)$$

The kinetic energy (K) and electrostatic potential energy (U) of the electron in hydrogen atom are

$$K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r} \text{ and } U = -\frac{e^2}{4\pi\varepsilon_0 r}$$

(The negative sign in U signifies that the electrostatic force is in the $-r$ direction.) Thus the total energy E of the electron in a hydrogen atom is

$$E = K + U = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$= -\frac{e^2}{8\pi\varepsilon_0 r} \qquad (12.4)$$

The total energy of the electron is negative. This implies the fact that the electron is bound to the nucleus. If E were positive, an electron will not follow a closed orbit around the nucleus.

Example 12.3 It is found experimentally that 13.6 eV energy is required to separate a hydrogen atom into a proton and an electron. Compute the orbital radius and the velocity of the electron in a hydrogen atom.

Solution Total energy of the electron in hydrogen atom is –13.6 eV = $-13.6 \times 1.6 \times 10^{-19}$ J = -2.2×10^{-18} J. Thus from Eq. (12.4), we have

$$E = -\frac{e^2}{8\pi\varepsilon_0 r} = -2.2 \times 10^{-18} \text{ J}$$

This gives the orbital radius

$$r = -\frac{e^2}{8\pi\varepsilon_0 E} = -\frac{(9\times10^9 \text{ N m}^2/\text{C}^2)(1.6\times10^{-19}\text{C})^2}{(2)(-2.2\times10^{-18}\text{ J})}$$

$= 5.3 \times 10^{-11}$ m.

The velocity of the revolving electron can be computed from Eq. (12.3) with $m = 9.1 \times 10^{-31}$ kg,

$$v = \frac{e}{\sqrt{4\pi\varepsilon_0 m r}} = 2.2\times10^6 \text{ m/s}.$$

EXAMPLE 12.3

12.3 Atomic Spectra

As mentioned in Section 12.1, each element has a characteristic spectrum of radiation, which it emits. When an atomic gas or vapour is excited at low pressure, usually by passing an electric current through it, the emitted radiation has a spectrum which contains certain specific wavelengths only. A spectrum of this kind is termed as emission line spectrum and it

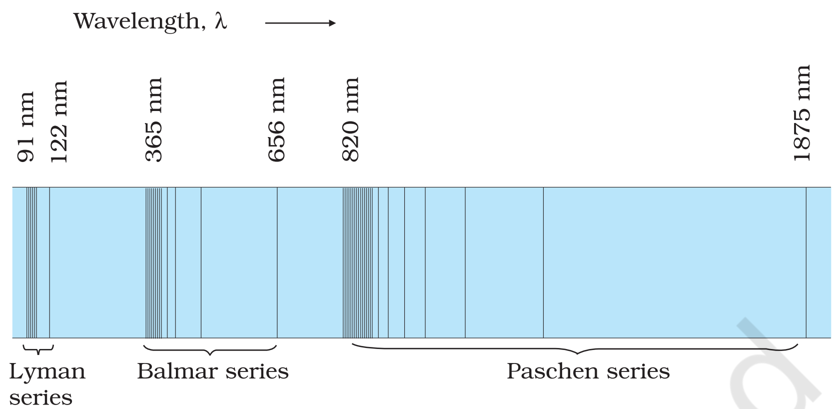


FIGURE 12.5 Emission lines in the spectrum of hydrogen.

consists of bright lines on a dark background. The spectrum emitted by atomic hydrogen is shown in Fig. 12.5. Study of emission line spectra of a material can therefore serve as a type of "fingerprint" for identification of the gas. When white light passes through a gas and we analyse the transmitted light using a spectrometer we find some dark lines in the spectrum. These dark lines correspond precisely to those wavelengths which were found in the emission line spectrum of the gas. This is called the *absorption spectrum* of the material of the gas.

12.3.1 Spectral series

We might expect that the frequencies of the light emitted by a particular element would exhibit some regular pattern. Hydrogen is the simplest atom and therefore, has the simplest spectrum. In the observed spectrum, however, at first sight, there does not seem to be any resemblance of order or regularity in spectral lines. But the spacing between lines within certain sets of the hydrogen spectrum decreases in a regular way (Fig. 12.5). Each of these sets is called a *spectral series*. In 1885, the first such series was observed by a Swedish school teacher Johann Jakob Balmer (1825–1898) in the visible region of the hydrogen spectrum. This series is called *Balmer series* (Fig. 12.6). The line with the longest wavelength, 656.3 nm in the red is called H_α; the next line with wavelength 486.1 nm in the blue-green is called H_β, the third line 434.1 nm in the violet is called H_γ; and so on. As the wavelength decreases, the lines appear closer together and are weaker in intensity.

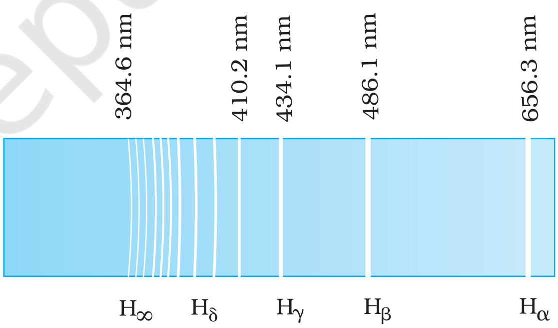


FIGURE 12.6 Balmer series in the emission spectrum of hydrogen.

Balmer found a simple empirical formula for the observed wavelengths

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad (12.5)$$

where λ is the wavelength, R is a constant called the *Rydberg constant*, and n may have integral values 3, 4, 5, etc. The value of R is $1.097 \times 10^7\ m^{-1}$. This equation is also called Balmer formula.

Taking $n = 3$ in Eq. (12.5), one obtains the wavelength of the H_α line:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2}\right) m^{-1}$$

$$= 1.522 \times 10^6\ m^{-1}$$

i.e., $\lambda = 656.3$ nm

For $n = 4$, one obtains the wavelength of H_β line, etc. For $n = \infty$, one obtains the limit of the series, at $\lambda = 364.6$ nm. This is the shortest wavelength in the Balmer series. Beyond this limit, no further distinct lines appear, instead only a faint continuous spectrum is seen.

Other series of spectra for hydrogen were subsequently discovered. These are known, after their discoverers, as Lyman, Paschen, Brackett, and Pfund series. These are represented by the formulae:

Lyman series:

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad n = 2,3,4... \tag{12.6}$$

Paschen series:

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \qquad n = 4,5,6... \tag{12.7}$$

Brackett series:

$$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right) \qquad n = 5,6,7... \tag{12.8}$$

Pfund series:

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right) \qquad n = 6,7,8... \tag{12.9}$$

The Lyman series is in the ultraviolet, and the Paschen, Brackett, and Pfund series are in the infrared region.

The Balmer formula Eq. (12.5) may be written in terms of frequency of the light, recalling that

$$c = \nu\lambda$$

or $$\frac{1}{\lambda} = \frac{\nu}{c}$$

Thus, Eq. (12.5) becomes

$$\nu = Rc\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \tag{12.10}$$

There are only a few elements (hydrogen, singly ionised helium, and doubly ionised lithium) whose spectra can be represented by simple formula like Eqs. (12.5) – (12.9).

Equations (12.5) – (12.9) are useful as they give the wavelengths that hydrogen atoms radiate or absorb. However, these results are empirical and do not give any reasoning why only certain frequencies are observed in the hydrogen spectrum.

12.4 Bohr Model of the Hydrogen Atom

The model of the atom proposed by Rutherford assumes that the atom, consisting of a central nucleus and revolving electron is stable much like sun-planet system which the model imitates. However, there are some fundamental differences between the two situations. While the planetary system is held by gravitational force, the nucleus-electron system being charged objects, interact by Coulomb's Law of force. We know that an