Procedure to form a differential equatition that will represent a given family of curves

(a) If the given family F_1 of curves depends on only one parameter then it is represented by an equation of the form

$$F_1(x, y, a) = 0 \quad — (1)$$

For example, the family of parabolas $y^2 = ax$ can be represented by an equation of the form $f(x, y, a): y^2 = ax$.

Differentiating equation (1) with respect to x, we get an equation involving y', y, x and a, i.e.

$$g(x, y, y', a) = 0 \quad — (2)$$

The required differential equation is then obtained by eliminating a from equations (1) and (2) as

$$F(x, y, y') = 0 \qquad (3)$$

(b) If the given family F_2 of curves depends on the parameters a, b (say) then it is represented by an equation of the form

$$F_2(x, y, a, b) = 0 \qquad (4)$$

Differentiating equation (4) with respect to x, we get an equation involving y', x, y, a, b, i.e.,

$$g(x, y, y', a, b) = 0 \qquad \text{—— (5)}$$

But it is not possible to eliminate two parameters a and b from the two equations, and so, we need a third equation. This equation is obtained by differentiating equation (5), with respect to x, to obtain a relation of the form

$$h(x, y, y', y'', a, b) = 0 \qquad \text{—— (6)}$$

The required differential equation is then obtained by eliminating a and b from equations (4), (5) and (6) as

$$F(x, y, y', y'') = 0 \qquad \text{—— (7)}$$