(Q) OF the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α only E_2 occurs is β and only E_3 occurs is γ. let the probability p that none of the events E1, E2 or E3 occurs satisfy the equations $(\alpha-2\beta)p = \alpha\beta$ and $(\beta-3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval $(0,1)$. Then $\frac{\text{Probability of occurrence of E1}}{\text{Probability of occurrence of E3}} =$

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- $P(E_1) = P_1$, $P(E_2) = P_2$, $P(E_3) = P_3$

given that, $P_1(1-P_2)(1-P_3) = \alpha$ —— ①

$P_2(1-P_1)(1-P_3) = \beta$ —— ②

$P_3(1-P_1)(1-P_2) = \gamma$ —— ③

and $(1-P_1)(1-P_2)(1-P_3) = P$ ——— (4)

$\Rightarrow \frac{P_1}{1-P_1} = \frac{\alpha}{P}$, $\frac{P_2}{1-P_2} = \frac{\beta}{P}$ & $\frac{P_3}{1-P_3} = \frac{\gamma}{P}$

Also, $\beta = \frac{\alpha P}{\alpha + 2P} = \frac{3\gamma P}{P - 2\gamma}$

$\Rightarrow \alpha P - 2\alpha\gamma = 3\alpha\gamma + 6P\gamma$

$\Rightarrow \frac{P_1}{1-P_1} - \frac{6P_3}{1-P_3} = \frac{5P_1P_3}{(1-P_1)(1-P_3)}$

$\Rightarrow P_1 - 6P_3 = 0$

$\frac{P_1}{P_3} = 6$