

(a) Let A & B be two events, then

(i) $P(A) + P(\bar{A}) = 1$

(ii) $P(A + B) = 1 - P(\bar{A}\bar{B})$

(iii) $P(A|B) = \frac{P(AB)}{P(B)}$

(iv) $P(A + B) = P(AB) + P(\bar{A}B) + P(A\bar{B})$

(v) $A \subset B \Rightarrow P(A) \leq P(B)$

(vi) $P(\bar{A}B) = P(B) - P(AB)$

(vii) $P(AB) \leq P(A)P(B) \leq P(A+B) \leq P(A) + P(B)$

(vii)

$P(AB) = P(A) + P(B) - P(A+B)$

(viii)

$P(AB) = P(A\bar{B}) + P(\bar{A}B)$

(ix)

$P(\text{Exactly one event}) = P(A\bar{B}) + P(\bar{A}B)$
 $= P(A) + P(B) - 2P(AB) = P(A+B) - P(AB)$

$$(x) P(\text{neither } A \text{ nor } B) = P(\bar{A}\bar{B}) = 1 - P(A+B)$$

$$(xi) P(\bar{A} + \bar{B}) = 1 - P(AB)$$

(b) Number of exhaustive cases of tossing n coins simultaneously (or of tossing a coin n times) $= 2^n$

(c) Number of exhaustive cases of throwing n dice simultaneously (or throwing one die n times) $= 6^n$

(d) Playing cards

(i) Total cards : 52 (26 red, 26 black)

(ii) Four suits : Heart, Diamond, Spade, Club - 13 cards each

(iii) Court cards : 12 (4 kings, 4 queens, 4 jacks)

(iv) Honour cards : 16 (4 aces, 4 kings, 4 queens, 4 jacks)

(e) Probability regarding n letters & their envelopes:
If n letters are placed into n directed envelopes at random, then

(i) Probability that all letters are in right envelopes $= \frac{1}{n!}$

(ii) Probability that all letters are not in right envelopes $= 1 - \frac{1}{n!}$

(iii) Probability that no letter is in right envelope $= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots + (-1)^n \frac{1}{n!}$

(iv) Probability that exactly r letters are in right envelopes

$$= \frac{1}{n!} \left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots + (-1)^{n-r} \frac{1}{(n-r)!} \right)$$