Example 15 One mapping (function) is selected at random from all the mappings of the set A = {1, 2, 3, ..., n} into itself. The probability that the mapping selected is one to one is

(A) $\frac{1}{n^n}$ (B) $\frac{1}{\lfloor n}$ (C) $\frac{\lfloor n-1}{n^{n-1}}$ (D) none of these

Solution (C) is the correct answer. Total number of mappings from a set A having n elements onto itself is n^n

Now, for one to one mapping the first element in A can have any of the n images in A; the 2[nd] element in A can have any of the remaining $(n-1)$ images, counting like this, the n^{th} element in A can have only 1 image.

Therefore, the total number of one to one mappings is $\lfloor n$.

Hence the required probability is $\frac{\lfloor n}{n^n} = \frac{n\lfloor n-1}{n\,n^{n-1}} = \frac{\lfloor n-1}{n^{n-1}}$.

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