Example 3

(a) How many two-digit positive integers are multiples of 3?

(b) What is the probability that a randomly chosen two-digit positive integer is a multiple of 3?

Solution

(a) 2 digit positive integers which are multiples of 3 are 12, 15, 18, ... , 99. Thus, there are 30 such integers.

(b) 2-digit positive integers are 10, 11, 12, ... , 99. Thus, there are 90 such numbers. Since out of these, 30 numbers are multiple of 3, therefore, the probability that a randomly chosen positive 2-digit integer is a multiple of 3, is $\frac{30}{90} = \frac{1}{3}$.

Example 4 A typical PIN (personal identification number) is a sequence of any four symbols chosen from the 26 letters in the alphabet and the ten digits. If all PINs are equally likely, what is the probability that a randomly chosen PIN contains a repeated symbol?

Solution A PIN is a sequence of four symbols selected from 36 (26 letters + 10 digits) symbols.

By the fundamental principle of counting, there are $36 \times 36 \times 36 \times 36 = 36^4 = 1{,}679{,}616$ PINs in all. When repetition is not allowed the multiplication rule can be applied to conclude that there are

$$36 \times 35 \times 34 \times 33 = 1{,}413{,}720 \text{ different PINs}$$

The number of PINs that contain at least one repeated symbol

$= 1{,}679{,}616 - 1{,}413{,}720 = 2{,}65{,}896$

Thus, the probability that a randomly chosen PIN contains a repeated symbol is

$$\frac{265,896}{1,679,616} = .1583$$

Example 5 An experiment has four possible outcomes A, B, C and D, that are mutually exclusive. Explain why the following assignments of probabilities are not permissible:

(a) $P(A) = .12$, $P(B) = .63$, $P(C) = 0.45$, $P(D) = -0.20$

(b) $P(A) = \frac{9}{120}$, $P(B) = \frac{45}{120}$ $P(C) = \frac{27}{120}$ $P(D) = \frac{46}{120}$

Solution

(a) Since $P(D) = -0.20$, this is not possible as $0 \le P(A) \le 1$ for any event A.

(b) $P(S) = P(A \cup B \cup C \cup D) = \frac{9}{120} + \frac{45}{120} + \frac{27}{120} + \frac{46}{120} = \frac{127}{120} \neq 1.$

This violates the condition that $P(S) = 1$.

Example 6 Probability that a truck stopped at a roadblock will have faulty brakes or badly worn tires are 0.23 and 0.24, respectively. Also, the probability is 0.38 that a truck stopped at the roadblock will have faulty brakes and/or badly working tires. What is the probability that a truck stopped at this roadblock will have faulty breaks as well as badly worn tires?

Solution Let B be the event that a truck stopped at the roadblock will have faulty brakes and T be the event that it will have badly worn tires. We have $P(B) = 0.23$, $P(T) = 0.24$ and $P(B \cup T) = 0.38$

and $P(B \cup T) = P(B) + P(T) - P(B \cap T)$

So $0.38 = 0.23 + 0.24 - P(B \cap T)$

$\Rightarrow$ $P(B \cap T) = 0.23 + 0.24 - 0.38 = 0.09$

Example 7 If a person visits his dentist, suppose the probability that he will have his teeth cleaned is 0.48, the probability that he will have a cavity filled is 0.25, the probability that he will have a tooth extracted is 0.20, the probability that he will have a teeth cleaned and a cavity filled is 0.09, the probability that he will have his teeth cleaned and a tooth extracted is 0.12, the probability that he will have a cavity filled and a tooth extracted is 0.07, and the probability that he will have his teeth cleaned, a cavity filled, and a tooth extracted is 0.03. What is the probability that a person visiting his dentist will have atleast one of these things done to him?

Solution Let C be the event that the person will have his teeth cleaned and F and E be the event of getting cavity filled or tooth extracted, respectively. We are given

$$P(C) = 0.48,\ P(F) = 0.25,\quad P(E) = .20,\quad P(C \cap F) = .09,$$

$$P(C \cap E) = 0.12,\ P(E \cap F) = 0.07 \text{ and } P(C \cap F \cap E) = 0.03$$

Now,
$$\begin{aligned} P(C \cup F \cup E) &= P(C) + P(F) + P(E) \\ &\quad - P(C \cap F) - P(C \cap E) - P(F \cap E) \\ &\quad + P(C \cap F \cap E) \\ &= 0.48 + 0.25 + 0.20 - 0.09 - 0.12 - 0.07 + 0.03 \\ &= 0.68 \end{aligned}$$

Long Answer Type

Example 8 An urn contains twenty white slips of paper numbered from 1 through 20, ten red slips of paper numbered from 1 through 10, forty yellow slips of paper numbered from 1 through 40, and ten blue slips of paper numbered from 1 through 10. If these 80 slips of paper are thoroughly shuffled so that each slip has the same probability of being drawn. Find the probabilities of drawing a slip of paper that is

(a) blue or white

(b) numbered 1, 2, 3, 4 or 5

(c) red or yellow and numbered 1, 2, 3 or 4

(d) numbered 5, 15, 25, or 35;

(e) white and numbered higher than 12 or yellow and numbered higher than 26.

Solution

(a) P (Blue or White) = P (Blue) + P (White) (Why?)

$$= \frac{10}{80} + \frac{20}{80} = \frac{30}{80} = \frac{3}{8}$$

(b) P (numbered 1, 2, 3, 4 or 5)

= P (1 of any colour) + P (2 of any colour)

+ P (3 of any colour) + P (4 of any colour) + P (5 of any colour)

$$= \frac{4}{80} + \frac{4}{80} + \frac{4}{80} + \frac{4}{80} + \frac{4}{80} = \frac{20}{80} = \frac{2}{8} = \frac{1}{4}$$

(c) P (Red or yellow and numbered 1, 2, 3 or 4)

= P (Red numbered 1, 2, 3 or 4) + P (yellow numbered 1, 2, 3 or 4)

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$$= \frac{4}{80} + \frac{4}{80} = \frac{8}{80} = \frac{1}{10}$$

(d) P (numbered 5, 15, 25 or 35)

= P (5) + P (15) + P (25) + P (35)

= P (5 of White, Red, Yellow, Blue) + P (15 of White, Yellow) + P (25 of Yellow)

+ P (35 of Yellow)

$$= \frac{4}{80} + \frac{2}{80} + \frac{1}{80} + \frac{1}{80} = \frac{8}{80} = \frac{1}{10}$$

(e) P (White and numbered higher than 12 or Yellow and numbered higher than 26)

= P (White and numbered higher than 12)

+ P (Yellow and numbered higher than 26)

$$= \frac{8}{80} + \frac{14}{80} = \frac{22}{80} = \frac{11}{40}$$