2) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$. Find A^{-1}

Using A^{-1}, solve the system of linear equations $x-2y=10$, $2x-y-z=8$, $-2y+z=7$

Solution: We have, $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

cofactors are: $A_{11} = -3$, $A_{12} = 2$ $A_{13} = 2$

$A_{21} = -2$, $A_{22} = 1$, $A_{23} = 1$

$A_{31} = -4$, $A_{32} = 2$, $A_{33} = 3$

$\therefore \operatorname{adj} A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

$|A| = 1(-3) - 2(-2) + 0 = 1$

$\therefore A^{-1} = \dfrac{\operatorname{adj} A}{|A|} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

Now the ~~st~~ system of linear equations is

$x-2y=10$, $2x-y-z=8$. and $-2y+z=7$

or $AX = B$

i.e. $\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$\therefore X = A^{-1} B$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} -30+16+14 \\ -20+8+7 \\ -40+16+21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$

$\therefore x = 0$, $y = -5$ and $z = -3$