If a is an integer lying in [−5, 30], then the probability that the graph $y = x^2 + 2(a + 4)x - 5a + 64$ is strictly above the x-axis is __________.

Solution:

$x^2 + 2(a + 4)x - 5a + 64 \geq 0$

If $D \leq 0$, then $(a + 4)^2 - (-5a + 64) < 0$ Or

$a^2 + 13a - 48 < 0$ Or

$(a + 16)(a - 3) < 0$

$\Rightarrow -16 < a < 3 \Leftrightarrow -5 \leq a \leq 2$

Then, the favourable cases are equal to the number of integers in the interval [−5, 2], i.e., 8.

The total number of cases is equal to the number of integers in the interval [−5, 30], i.e., 36.

Hence, the required probability is $8 / 36 = 2 / 9$.