

Forty teams play a tournament. Each team plays with every other team just once. Each game results in a win for one team. If each team has a 50% chance of winning each game, the probability that at the end of the tournament, every team has won a different number of games is _____.

Solution:

Team totals must be 0, 1, 2, 39.

Let the teams be T_1, T_2, \dots, T_{40} , so that T_i loses to T_j for $i < j$. In other words, this order uniquely determines the result of every game. There are $40!$ such orders and 780 games, so 2^{780} possible outcomes for the games.

Hence, the probability is $40! / 2^{780}$.