

Let  $M$  be a  $3 \times 3$  matrix satisfying

$$M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \quad M \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \quad \text{and} \quad M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}. \quad \text{Then the}$$

sum of the diagonal entries of  $M$  is:

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Solution: (9).

$$\text{Let } M = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$\text{then } \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow b_1 = -1, b_2 = 2, b_3 = 3$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\Rightarrow a_1 - b_1 = 1, a_2 - b_2 = 1, a_3 - b_3 = -1$$

$$\Rightarrow a_1 = 0, a_2 = 3, a_3 = 2$$

$$\text{and } \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

$$\Rightarrow a_3 + b_3 + c_3 = 12 \Rightarrow c_3 = 7 \quad [\because a_3 = 2, b_3 = 3]$$

$$\therefore \text{sum of diagonal elements} = a_1 + b_2 + c_3 = 0 + 2 + 7 = \boxed{9}$$