

3) Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then the number of 3×3 matrices B

with entries from the set $\{1, 2, 3, 4, 5\}$ and satisfying

$AB = BA$ is _____.

[Main July 22, 2021 (II)]

Solution: (3125)

$$\text{Let } B = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$$

$$\therefore AB = BA$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} y_1 & y_2 & y_3 \\ x_1 & x_2 & x_3 \\ z_1 & z_2 & z_3 \end{bmatrix} = \begin{bmatrix} x_2 & x_1 & x_3 \\ y_2 & y_1 & y_3 \\ z_2 & z_1 & z_3 \end{bmatrix}$$

$$\Rightarrow y_1 = x_2, y_2 = x_1, y_3 = x_3, z_1 = z_2$$

$$\therefore B = \begin{bmatrix} x_1 & x_2 & x_3 \\ x_2 & x_1 & x_3 \\ z_1 & z_1 & z_3 \end{bmatrix}$$

\therefore 5 elements can be written independently in

$$5^5 = 3125 \text{ ways}$$

\therefore Number of Matrices = 3125