14. If the letters of the word ASSASSINATION are arranged at random. Find the Probability that

(a) Four S's come consecutively in the word

(b) Two I's and two N's come together

(c) All A's are not coming together

(d) No two A's are coming together.

Sol. We have word 'ASSASSINATION'.

Number of letters = 13

Letters are $3A$'s, $4S$'s, $2I$'s, $2N$'s, 1 T's and 1 O's

Total number of ways these letters can be arranged = $n(S) = \dfrac{13!}{3!4!2!2!}$

(a) If for S's come consecutively in the word, then we considers these 4 S's as 1 group.

So, now number of letters is 10 i.e., $(SSSS), A, A, A, I, I, N, N, T, O$

$\therefore \qquad n(E) = \dfrac{10!}{3!2!2!}$

$\therefore \qquad$ Required probability $= \dfrac{\dfrac{10!}{3!2!2!}}{\dfrac{13!}{3!4!2!2!}} = \dfrac{4!}{13\times12\times11} = \dfrac{2}{143}$

(b) If 2 I's and $2N$'s come together, then there as 10 alphabets.

i.e., (IINN), $A, A, A, S, S, S, S, T, O$

Number of words when 2 I's and 2 N's are come together

$= \dfrac{10!}{3!4!} \times \dfrac{4!}{2!2!}$

$\therefore \qquad$ Required probability $= \dfrac{\dfrac{10!4!}{3!4!2!2!}}{\dfrac{13!}{3!4!2!2!}} = \dfrac{4!}{13\times12\times11} = \dfrac{2}{143}$

(c) If all A's are coming together, then there are 11 alphabets

i.e., $(AAA), S, S, S, S, I, I, N, N, T, O$

$\therefore$ Number of words when all A's come together $= \dfrac{11!}{4!2!2!}$

$\therefore$ Probability when all A's come together

$$= \frac{\dfrac{11!}{4!2!2!}}{\dfrac{13!}{4!3!2!2!}} = \frac{3!}{13\times 12} = \frac{1}{26}$$

Then the probability that all A's does not come together

$$= 1 - \frac{1}{26} = \frac{25}{26}$$

(d) If no two A's are together, then first we arrange the alphabets other than A's.

i.e. $S, S, S, S, I, I, N, N, T, O$

These letters can be arranged in $\dfrac{10!}{4!2!2!}$ ways.

$$\times S \times S \times S \times S \times I \times I \times N \times N \times T \times O \times$$

Arrangement of these letters creates eleven gaps shown as '×'

Three gaps for three A's can be selected in ${}^{11}C_3$ ways.

$\therefore$ Total number of words when no two A's together

$$= {}^{11}C_3 \times \frac{10!}{4!2!2!} = \frac{11!}{3!8!} \times \frac{10!}{4!2!2!}$$

$\therefore$ The probability that no two A's come together

$$= \frac{\dfrac{11!\times 10!}{3!8!4!2!2!}}{\dfrac{13!}{4!3!2!2!}} = \frac{10!}{8!\times 13\times 12} = \frac{10\times 9}{13\times 12} = \frac{15}{26}$$