4) Solve: $\dfrac{dy}{dx} = \dfrac{\sin y + x}{\sin 2y - x\cos y}$

Solution: We have $\dfrac{dy}{dx} = \dfrac{\sin y + x}{\sin 2y - x\cos y}$

$$\therefore \cos y \frac{dy}{dx} = \frac{\sin y + x}{2\sin y - x} \qquad \text{——— (i)}$$

Put $\sin y = vx$

$$\therefore \cos y \frac{dy}{dx} = \cancel{\sin y}\, v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx + x}{2vx - x}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v+1}{2v-1} - v$$

$$\Rightarrow \int \frac{2v-1}{-2v^2+2v+1}\,dv = \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2}\log_e(-2v^2+2v+1) = \log_e x + \log_e c$$

$$\Rightarrow \log_e \frac{1}{\sqrt{-2v^2+2v+1}} = \log_e cx$$

$$\Rightarrow \frac{1}{\sqrt{-2v^2+2v+1}} = cx$$

$$\Rightarrow \frac{1}{\sqrt{-2\frac{\sin^2 y}{x^2} + 2\frac{\sin y}{x} + 1}} = cx$$

$$\Rightarrow -2\sin^2 y + 2x\sin y + x^2 = c_1$$