

3) The differential equation representing the family of ellipse having foci either on the x -axis or on the y -axis centre at the origin and passing through the point $(0,3)$ is:

solution: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

since, it passes through $(0,3)$

$$\therefore \frac{0}{a^2} + \frac{9}{b^2} = 1$$

$$\Rightarrow b^2 = 9$$

\therefore eq. of ellipse becomes:

$$\frac{x^2}{a^2} + \frac{y^2}{9} = 1$$

differentiating w.r.t. x , we get;

$$\frac{2x}{a^2} + \frac{2y}{9} \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{y}{x} \left(\frac{dy}{dx} \right) = \frac{-9}{a^2}$$

Again differentiating w.r.t. x , we get;

$$\frac{y}{x} \frac{d^2y}{dx^2} + \frac{x \frac{dy}{dx} - y}{x^2} \frac{dy}{dx} = 0 \Rightarrow \boxed{xyy'' + x(y')^2 - yy' = 0}$$