1) If $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$; find $y(x)$.

The given differential equation is:

$$y + \frac{d}{dx}(xy) = x(\sin x + \log x)$$

i.e. $\quad x\frac{dy}{dx} + 2y = x(\sin x + \log x)$

or $\quad \frac{dy}{dx} + \frac{2}{x}y = \sin x + \log x$

This is a linear differential equation

$$I.F. = e^{2\int \frac{1}{x}dx} = e^{2\log x} = x^2$$

Thus, solution is given by

$$yx^2 = \int x^2 (\sin x + \log x)dx + c$$

$$= -x^2\cos x + 2x\sin x + 2\cos x + \frac{x^3}{3}\log x - \frac{x^3}{9} + c$$

or $\quad y = -\cos x + \frac{2}{x}\sin x + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{3} + \frac{c}{x^2}$