Product of two vectors

Dot (scalar) product : $\vec{u}\cdot\vec{v}$

Cross (vector) product : $\vec{u}\times\vec{v}$

$\vec{u}\,\vec{v}$ is meaningless

$(\vec{u})(\vec{v})$ "

$2\vec{a}$ is scalar multiple of $\vec{a}$

$2\cdot\vec{a}$, $2\times\vec{a}$ are meaningless

$2(\vec{a})$ is a scalar multiplication of $\vec{a}$

Def : If $\vec{u}$ and $\vec{v}$ are non-zero vectors - then

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dot product of $\vec{u}$ and $\vec{v}$, denoted by $\vec{u}\cdot\vec{v}$ is defined by

$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$$

where θ is the angle b/n $\vec{u}$ & $\vec{v}$

$$\theta = \vec{u} \wedge \vec{v} \qquad 0 \le \theta \le \pi$$

if $\vec{u}=0$, or $\vec{v}=0$ we define $\vec{u}\cdot\vec{v}=0$ (θ is undefined)

$$\vec{0}\cdot\vec{v} = 0 \quad \forall \vec{v}$$

$$\vec{0}\cdot\vec{0} = 0$$

$\vec{u}\cdot\vec{v}$ is a scalar

$$|\vec{u}\cdot\vec{v}| \le |\vec{u}||\vec{v}| \qquad \because |\cos\theta| \le 1$$

$$|\vec{u}\cdot\vec{v}| = ||\vec{u}||\vec{v}|\cos\theta| = |\vec{u}||\vec{v}||\cos\theta|$$

$$\le |\vec{u}||\vec{v}|$$

$$-|\vec{u}||\vec{v}| \le \vec{u}\cdot\vec{v} \le |\vec{u}||\vec{v}|$$

$$\theta = \pi \qquad \qquad \theta = 0^\circ$$

If $\vec{u}$ & $\vec{v}$ are non-zero vectors

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$$

$\vec{u}$ and $\vec{v}$ are orthogonal $\Leftrightarrow$ $\vec{u}\cdot\vec{v} = 0$

($\vec{0}$ is orthogonal to every vector)

$$\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$$

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$\vec{u}\cdot\vec{u} = |\vec{u}|^2 = \vec{u}^2$

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

$\vec{u}, \vec{v}, \vec{w}$ be vectors and λ is a scalar

$\vec{u}\cdot(\vec{v}+\vec{w}) = \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w}$

$(\lambda\vec{u})\cdot\vec{v} = \vec{u}\cdot(\lambda\vec{v}) = \lambda\,\vec{u}\cdot\vec{v}$

$\vec{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$

$\vec{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$

$\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$

$\vec{u}\cdot\vec{u} = u_1^2 + u_2^2 + u_3^2$

$|\vec{u}|^2 = u_1^2 + u_2^2 + u_3^2$

$|\vec{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2}$

$|\vec{u}+\vec{v}|^2 = (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) = \vec{u}\cdot\vec{u} + \vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{u} + \vec{v}\cdot$

$= |\vec{u}|^2 + |\vec{v}|^2 + 2(\vec{u}\cdot\vec{v})$

$|\vec{u}\pm\vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 \pm 2(\vec{u}\cdot\vec{v})$

$\vec{a} = \hat{i} + 2\hat{j}$, $\hat{a} = \frac{\hat{i}+2\hat{j}}{\sqrt{3}}$

$\vec{b} = \hat{u} + 2\hat{v}$, $\hat{b} = \frac{\hat{u}+2\hat{v}}{\sqrt{3}}$ X

$|\hat{i}+2\hat{j}|^2 = |\hat{i}|^2 + |2\hat{j}|^2 + (2\hat{i})\cdot(2\hat{j})$

$= 1 + 4 + 0 = 5$

$|\hat{u}+2\hat{v}|^2 = |\hat{u}|^2 + |2\hat{v}|^2 + 2\,\hat{u}\cdot(2\hat{v})$

$= 1 + 4(1) + 4\,\hat{u}\cdot\hat{v} = 5 + 4\,\hat{u}\cdot\hat{v}$

$$|\vec{u}+\vec{v}+\vec{w}|^2 = |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2(\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{w} + \vec{u}\cdot\vec{w})$$

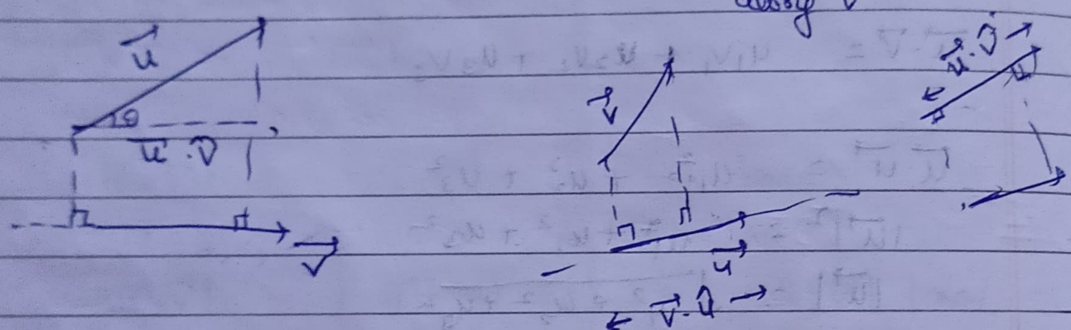
$$\vec{a}\cdot\vec{b} \le |\vec{a}||\vec{b}|$$

$$a_1b_1 + a_2b_2 + a_3b_3 \le \sqrt{a_1^2+a_2^2+a_3^2}\cdot\sqrt{b_1^2+b_2^2+b_3^2}$$

$$\boxed{a_1b_1 + a_2b_2 + a_3b_3 + \cdots + a_nb_n \le \sqrt{a_1^2+a_2^2+\cdots+a_n^2}\cdot\sqrt{b_1^2+b_2^2+\cdots+b_n^2}}$$

Cauchy Schwarz Inequality

$\vec{u}\cdot\hat{v} = |\vec{u}||\hat{v}|\cos\theta = |\vec{u}|\cos\theta =$ projection of $\vec{u}$ along $\vec{v}$



$$|\vec{v}|\cos\theta = \frac{|\vec{u}||\vec{v}|\cos\theta}{|\vec{u}|} = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|} = \frac{\vec{v}\cdot\vec{u}}{|\vec{u}|}$$
$$= \vec{v}\cdot\hat{u}$$

projection vector of $\vec{u}$ along $\vec{v}$ $= (\vec{u}\cdot\hat{v})\,\hat{v}$

$$= \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}\,\hat{v}$$

$$= \frac{\vec{u}\cdot\vec{v}\,\vec{v}}{|\vec{v}|^2}$$

eg $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$$\left(|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2\right)_{max} = ?$$

$$|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2$$
$$= 2(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2) - 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$$

$$= 2\left(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2\right) + |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 - |\vec{a}+\vec{b}+\vec{c}|^2$$

$$= 3\left(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2\right) - |\vec{a}+\vec{b}+\vec{c}|^2$$

$$\leq 3\left(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2\right)$$

$$= 9$$

$$|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 \leq 9$$

For equality $\quad |\vec{a}+\vec{b}+\vec{c}| = 0$

i.e. $\boxed{\vec{a}+\vec{b}+\vec{c} = \vec{0}}$