

In an infinite GP second term is x and its sum is 4, find the set of values of x .

- a) $x \in (0, 1)$ b) $x \in (-\infty, 1)$ c) $x \in (-8, 1)$
d) $x \in (-8, 1] - \{0\}$

Solution: -

Key ~~idea~~ concept: for an infinite GP to be defined, $|\text{ratio}| < 1 \Rightarrow -1 < \text{ratio} < 1$
now,

$$S_{\infty} = \frac{a}{1-r} = 4$$

$$\text{second term} = ar = x$$

$$\Rightarrow ar = x \Rightarrow a = \frac{x}{r}$$

$$\frac{a}{1-r} = 4$$

$$\Rightarrow \frac{x/r}{1-r} = 4$$

$$\Rightarrow \frac{x}{r-r^2} = 4$$

$$x = 4r - 4r^2$$

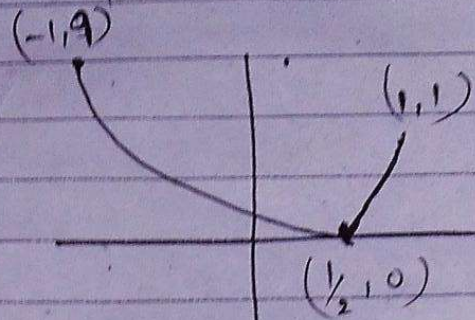
$$4r^2 - 4r = -x$$

$$4r^2 - 4r + 1 = 1 - x$$

$$\Rightarrow (2r-1)^2 = 1-x$$

~~no~~

graph of $y = (2x-1)^2$, $-1 < x < 1$



now,

range of $(2x-1)^2$ when $-1 < x < 1$ is $[0, 9)$

$$\therefore 0 \leq (2x-1)^2 < 9$$

$$0 \leq 1-x < 3$$

$$-1 \leq -x < 2$$

$$1 \geq x > -2$$

$$\Rightarrow x \in (-2, 1]$$

and $x \neq 0$ ($\because a \neq 0, r \neq 0$)

$$\therefore x \in (-2, 1] - \{0\}$$