

5) The curve amongst the family of curves represented by the differential equation, $(x^2 - y^2) dx + 2xy dy = 0$ which passes through $(1, 1)$ is: [Main Jan. 10, 2019 (II)]

- (a) a circle with centre on the x-axis.
- (b) an ellipse with major axis along the y-axis.
- (c) a circle with centre on the y-axis.
- (d) a hyperbola with transverse axis along the x-axis.

solution: (a): $(x^2 - y^2) dx + 2xy dy = 0$

$$y^2 dx - 2xy dy = x^2 dx$$

$$2xy dy - y^2 dx = -x^2 dx$$

$$d(xy^2) = -x^2 dx$$

$$\frac{x d(y^2) - y^2 d(x)}{x^2} = -dx$$

$$d\left(\frac{y^2}{x}\right) = -dx \int d\left(\frac{y^2}{x}\right) = -\int dx$$

$$\frac{y^2}{x} = -x + C \quad \dots (i)$$

Since, the above curve passes through the point $(1, 1)$

$$\text{Then, } \frac{1^2}{1} = -1 + C \Rightarrow C = 2$$

Now, the curve (i) becomes

$$y^2 = -x^2 + 2x$$

$$\Rightarrow y^2 = -(x-1)^2 + 1$$

$$(x-1)^2 + y^2 = 1$$

The above equation represents a circle with centre $(1, 0)$ and centre lies on the x-axis.