In a circuit, a metal filament lamp is connected in series with a capacitor of capacitance C μF across a 200 V, 50 Hz supply. The power consumed by the lamp is 500 W while the voltage drop across it is 100 V. Assume that there is no inductive load in the circuit. Take rms values of the voltages. The magnitude of the phase-angle (in degrees) between the current and the supply voltage is φ. Assume, $\pi\sqrt{3} \approx 5$.

The value of φ is _____.

4 & 5. According to question, equivalent figure is:-
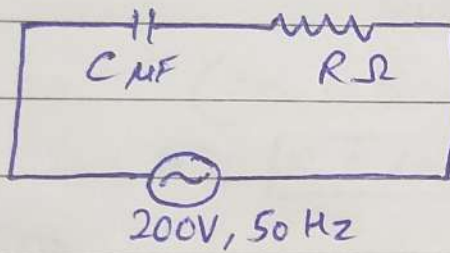
$$\nu = 50 \, Hz \Rightarrow \boxed{\omega = 2\pi\nu = 100\pi \, rad/s}$$

$$P_{lamp} = 500 W \quad, \quad \Delta V_{(across \, lamp)} = 100 \, V$$

$$\therefore \quad P = \frac{V^2}{R} \quad (where \; 'R' \; is \; resistance \; of \; lamp)$$

$$\Rightarrow 500 = \frac{(100)^2}{R}$$

$$\Rightarrow \boxed{R = 20 \, \Omega}$$

Also, $P_{lamp} = V_{lamp} \, I_{lamp}$

$$\Rightarrow \quad 500 = (100) \, I_{lamp}$$

$$\Rightarrow \boxed{I_{lamp} = 5 \, A}$$

As it is series circuit, then,

$$i_{lamp} = i_{source} = 5A$$

Now, $V_{source} = i_{source} Z$

$$\Rightarrow 200 = (5) Z$$

$$\Rightarrow \boxed{Z = 40 \, \Omega}$$

Now, $Z = \sqrt{R^2 + X_c^2}$

$$\Rightarrow 40 = \sqrt{400 + X_c^2}$$

$$\Rightarrow X_c^2 = 1200$$

$$\Rightarrow \frac{1}{\omega c} = 1200 \qquad X_c = 20\sqrt{3} \, \Omega$$

$$\Rightarrow \frac{1}{\omega c} = 20\sqrt{3}$$

$$\Rightarrow C = \frac{1}{100\pi \times 20\sqrt{3}}$$

(On taking $\sqrt{3}\pi \approx 5$)

$$\Rightarrow \boxed{C = 100 \mu F}$$

Also, $\cos\phi = \dfrac{R}{Z}$

$$= \frac{20}{40}$$

$$\cos\phi = \frac{1}{2}$$

$$\Rightarrow \boxed{\phi = 60°}$$