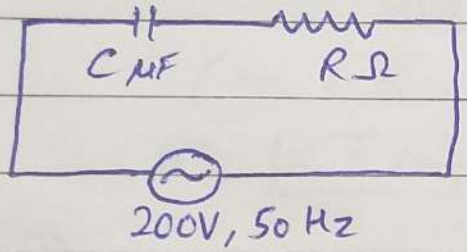
In a circuit, a metal filament lamp is connected in series with a capacitor of capacitance C μF across a 200 V, 50 Hz supply. The power consumed by the lamp is 500 W while the voltage drop across it is 100 V. Assume that there is no inductive load in the circuit. Take rms values of the voltages. The magnitude of the phase-angle (in degrees) between the current and the supply voltage is φ. Assume, $\pi\sqrt{3} \approx 5$.(JEE ADVANCED 2021)

The value of C is _____.

4 & 5. According to question, equivalent figure is:-



$\nu = 50\,Hz \Rightarrow \boxed{\omega = 2\pi\nu = 100\pi\ rad/s}$

$P_{lamp} = 500W$, $\Delta V_{(across\ lamp)} = 100V$

$\therefore P = \frac{V^2}{R}$ (where 'R' is resistance of lamp)

$\Rightarrow 500 = \frac{(100)^2}{R}$

$\Rightarrow \boxed{R = 20\,\Omega}$

Also, $P_{lamp} = V_{lamp}\, I_{lamp}$

$\Rightarrow 500 = (100)\, I_{lamp}$

$\Rightarrow \boxed{I_{lamp} = 5A}$

As it is series circuit, then,

$i_{lamp} = i_{source} = 5A$

Now, $V_{source} = i_{source} Z$

$\Rightarrow 200 = (5) Z$

$\Rightarrow \boxed{Z = 40\,\Omega}$

Now, $Z = \sqrt{R^2 + X_c^2}$

$\Rightarrow 40 = \sqrt{400 + X_c^2}$

$\Rightarrow X_c^2 = 1200$

$\Rightarrow X_c = 20\sqrt{3}\,\Omega$

$\Rightarrow \frac{1}{\omega C} = 20\sqrt{3}$

$\Rightarrow C = \frac{1}{100\pi \times 20\sqrt{3}}$

(On taking $\sqrt{3}\pi \approx 5$)

$\Rightarrow \boxed{C = 100\,\mu F}$

Also, $\cos\phi = \frac{R}{Z}$

$= \frac{20}{40}$

$\cos\phi = \frac{1}{2}$

$\Rightarrow \boxed{\phi = 60^\circ}$