Relations & Functions

Cartesian product of sets:

Given two non-empty sets P & Q. The cartesian product $P \times Q$ is the set of all ordered pairs of elements from P & Q i.e.

$$P \times Q = \{(p, q); p \in P; q \in Q\}$$

Relation

Let A & B be two non-empty sets. Then any subset 'R' of $A \times B$ is a relation from A to B.

If $(a, b) \in R$, then we write it as a R b which is read as a is related to b' by the relation R', 'b' is also called image of 'a' under R.

If R is a relation from A to B, then the set of first elements in R is called domain & the set of second elements in R is called range of R. symbolically.

$$\text{Domain of } R = \{x : (x, y) \in R\}$$

$$\text{Range of } R = \{y : (x, y) \in R\}$$

The set B is called co-domain of relation R.

Note that range $\subset$ co-domain.

Let A, B be two sets and let R be a relation from a set A to set B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by

$$R^{-1} = \{(b, a) : (a, b) \in R\}$$

Clearly, $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$

Also, Dom (R) = Range (R^{-1}) and Range (R) = Dom (R^{-1}).

Functions

A relation 'f' from a set A to set B is said to be a function if every element of set A has one and only one image in set B.

Domain:

When we define $y = f(x)$ with a formula and the domain is not stated explicitly, the domain is assumed to be the largest set of x–values for which the formula gives real y–values.

The domain of $y = f(x)$ is the set of all real x for which $f(x)$ is defined (real).

Points to determine Domain:

(i) Expression under even root (i.e. square root, fourth root etc.) should be non–negative.

(ii) Denominator $\neq 0$.

(iii) $\log_a x$ is defined when $x > 0$, $a > 0$ and $a \neq 1$.

(iv) If domain of $y = f(x)$ and $y = g(x)$ are D_1 and D_2 respectively, then the domain of $f(x) \pm g(x)$ or $f(x) . g(x)$ is $D_1 \cap D_2$. While domain of $\frac{f(x)}{g(x)}$ is $D_1 \cap D_2 - \{x: g(x) = 0\}$.

Range:

The set of all f-images of elements of A is known as the range of f & denoted by $f(A)$.

Range $= f(A) = \{f(x) : x \in A\}$;

$f(A) \subseteq B$ {Range $\subseteq$ Co-domain}.

Points to determine range:

First of all find the domain of $y = f(x)$

(i) If domain $\in$ finite number of points

$\Rightarrow$ range $\in$ set of corresponding $f(x)$ values.

(ii) If domain $\in$ R or R $-$ {some finite points}

Put $y = f(x)$

Then express x in terms of y. From this find y for x to be defined. (i.e., find the values of y for which x exists).

(iii) If domain $\in$ a finite interval, find the least and greater value for range using monotonocity.

Conditions of not being function

1) If not all elements of set A are associated with some elements of set B then it is not function

2) An element of set A is not associated with a unique element of set B

Conditions for one-one mapping

1) If $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$, then $f(x)$ is one-one.

2) A function is one-one, iff no line paralle to x-axis meets the graph of function at more than one point.

3) For checking whether $f(x)$ is One-One find whether function is only increasing or only decreasing in their domain. If yes, then function is one-one, i.e. if $f'(x) \geq 0, \forall x \in$ domain or i.e. if $f'(x) \leq 0, \forall x \in$ domain, then function is one-one

Condition for onto mapping

1) Consider two sets A and B. If for every element of B, there is at least one or more than one element matching with A, then the function is called onto function.

2) We can define onto function as if any function states surjection by limit its codomain to its range.
The domain is basically what can go into the function, codomain states possible outcomes and range denotes the actual input of the function.
Every onto function has a right inverse.
Every function with a right inverse is a surjective function.
If we compose onto functions, it will result in onto function only.