# Relations and Functions

**Relation :**

If A and B are two non-empty sets, then a relation R from A to B is a subset of A x B.

If R ⊆ A x B and (a, b) ∈ R, then we say that a is related to b by the relation R, written as aRb.

Let R be a relation from a set A to set B. Then, set of all first components or coordinates of the ordered pairs belonging to R is called : the domain of R, while the set of all second components or coordinates = of the ordered pairs belonging to R is called the range of R.

Thus, domain of R = {a : (a , b) ∈ R} and range of R = {b : (a, b) ∈ R}

**Inverse Relation**

If A and B are two non-empty sets and R be a relation from A to B, such that R = {(a, b) : a ∈ A, b ∈ B}, then the inverse of R, denoted by $R^{-1}$ , i a relation from B to A and is defined by $R^{-1}$ = {(b, a) : (a, b) ∈ R}

**Equivalence Classes of an Equivalence Relation**

Let R be equivalence relation in A (≠ Φ). Let a ∈ A.

Then, the equivalence class of a denoted by [a] or {a} is defined as the set of all those points of A which are related to a under the relation R.

**Composition of Relation**

Let R and S be two relations from sets A to B and B to C respectively, then we can define relation SoR from A to C such that (a, c) ∈ So R ⇔ ∃ b ∈ B such that (a, b) ∈ R and (b, c) ∈ S.

This relation SoR is called the composition of R and S.

(i) RoS ≠ SoR
(ii) $(SoR)^{-1} = R^{-1}oS^{-1}$ known as **reversal rule.**

**Congruence Modulo m**

Let m be an arbitrary but fixed integer. Two integers a and b are said to be congruence modulo m, if a – b is divisible by m and we write a ≡ b (mod m).

i.e., a ≡ b (mod m) ⇔ a – b is divisible by m.

**Important Results on Relation**

- If R and S are two equivalence relations on a set A, then R ∩ S is also on ‘equivalence relation on A.
- The union of two equivalence relations on a set is not necessarily an equivalence relation on the set.
- If R is an equivalence relation on a set A, then $R^{-1}$ is also an equivalence relation on A.
- If a set A has n elements, then number of reflexive relations from A to A is $2^{n2 - 2}$ ⊠ Let A and B be two non-empty finite sets consisting of m and n elements, respectively. Then, A x B consists of mn ordered pairs. So, total number of relations from A to B is $2^{nm}$.

**Properties**

- Generally binary operations are represented by the symbols * , +, … etc., instead of letters figure etc.
- Addition is a binary operation on each one of the sets N, Z, Q, R and C of natural numbers, integers, rationals, real and complex numbers, respectively. While addition on the set S of all irrationals is not a binary operation.
- Multiplication is a binary operation on each one of the sets N, Z, Q, R and C of natural numbers, integers, rationals, real and complex numbers, respectively. While multiplication on the set S of all irrationals is not a binary operation.
- Subtraction is a binary operation on each one of the sets Z, Q, R and C of integers, rationals, real and complex numbers, respectively. While subtraction on the set of natural numbers is not a binary operation.
- Let S be a non-empty set and P(S) be its power set. Then, the union and intersection on P(S) is a binary operation.

- Division is not a binary operation on any of the sets N, Z, Q, R and C. However, it is not a binary operation on the sets of all non-zero rational (real or complex) numbers.

- Exponential operation (a, b) → $a^b$ is a binary operation on set N of natural numbers while it is not a binary operation on set Z of integers.

**Types of Binary Operations**

**(i) Associative Law** A binary operation * on a non-empty set S is said to be associative, if (a * b) * c = a * (b * c), ∀ a, b, c ∈ S.

Let R be the set of real numbers, then addition and multiplication on R satisfies the associative law.

**(ii) Commutative Law** A binary operation * on a non-empty set S is said to be

commutative, if a * b = b * a, ∀ a, b ∈ S.

Addition and multiplication are commutative binary operations on Z but subtraction not a commutative binary operation, since

2 — 3 ≠ 3— 2 .

Union and intersection are commutative binary operations on the power P(S) of all subsets of set S. But difference of sets is not a commutative binary operation on P(S).

**(iii) Distributive Law** Let * and o be two binary operations on a non-empty sets. We say that * is distributed over o., if

a * (b o c)= (a * b) o (a * c), ∀ a, b, c ∈ S also called (left distribution) and (b o c) * a = (b * a) o (c * a), ∀ a, b, c ∈ S also called (right distribution).

Let R be the set of all real numbers, then multiplication distributes addition on R.

Since, a.(b + c) = a.b + a.c,∀ a, b, c ∈ R.

(iv) Identity Element Let * be a binary operation on a non-empty set S. An element e a S,

if it exist such that a * e = e * a = a, ∀ a ∈ S. is called an identity elements of S, with

respect to *.

For addition on R, zero is the identity elements in R.

Since, a + 0 = 0 + a = a, ∀ a ∈ R

For multiplication on R, 1 is the identity element in R.

Since, a x 1 =1 x a = a,∀ a ∈ R

Let P (S) be the power set of a non-empty set S. Then, Φ is the identity element for union on P
(S) as

A ∪ Φ =Φ ∪ A = A, ∀ A ∈ P(S)

Also, S is the identity element for intersection on P(S).

Since, A ∩ S=A ∩ S=A, ∀ A ∈ P(S).

For addition on N the identity element does not exist. But for multiplication on N the idenitity element is 1.

(v) Inverse of an Element Let * be a binary operation on a non-empty set 'S' and let 'e' be the identity element.

Let a ∈ S. we say that $a^{-1}$ is invertible, if there exists an element b ∈ S such that a * b = b * a = e

Also, in this case, b is called the inverse of a and we write, $a^{-1}$ = b

Addition on N has no identity element and accordingly N has no invertible element.

Multiplication on N has 1 as the identity element and no element other than 1 is invertible.

Let S be a finite set containing n elements. Then, the total number of binary operations on S in nn2

Let S be a finite set containing n elements. Then, the total number of commutative binary operation on S is n [n(n+1)/2].