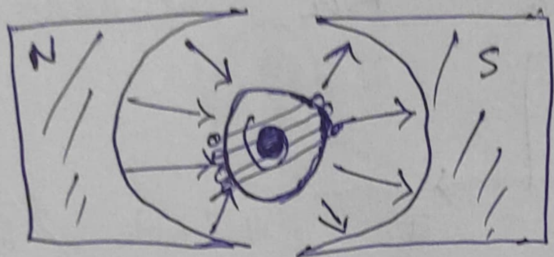
Moving Coil Galvanometer



N Turns

A : Area

$$\tau_{current} = \vec{m} \times \vec{B} = mB$$

$$= NIAB$$

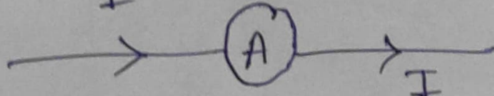
$\tau_{spring} = k\phi$ $\quad$ k : spring constant

ϕ : Angle of rotation

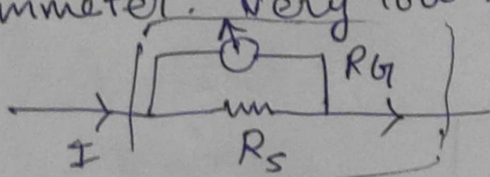
At equilibrium : $k\phi = NIAB$

$$\boxed{\phi = \frac{NAB}{k} \cdot I}$$

Galvanometer as Ammeter



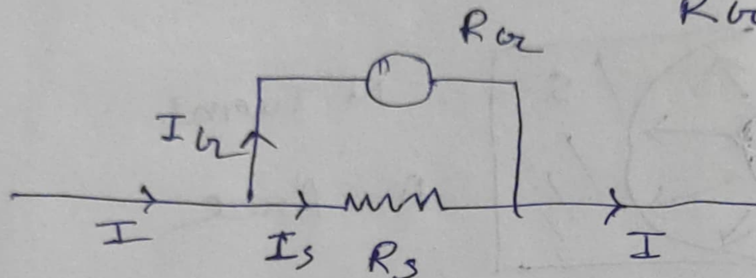
Ammeter : very low resistance



$$\frac{1}{R} = \frac{1}{R_G} + \frac{1}{R_s} = \frac{R_s + R_G}{R_s R_G}$$

$$R = \frac{R_s R_G}{R_s + R_G}$$

$$R_s <<< R_G \qquad R \simeq \frac{R_s R_G}{R_G} \simeq R_s$$



$$I = I_s + I_G$$

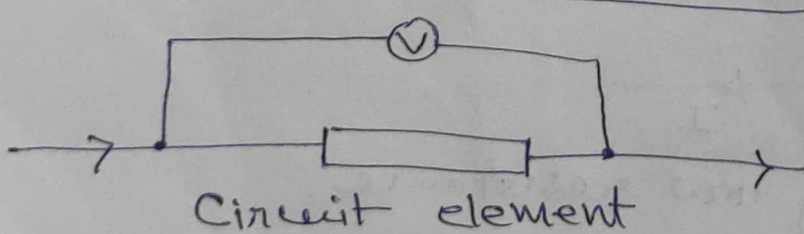
$$I_G R_G = I_s R_s$$

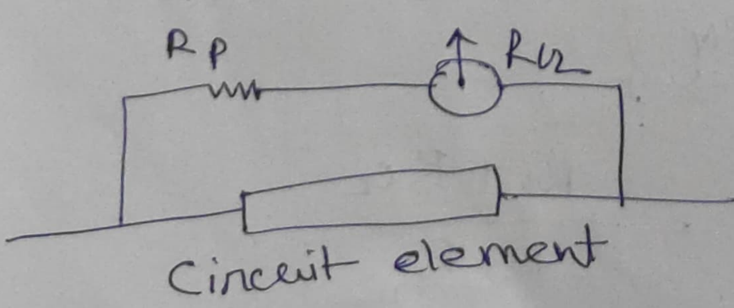
$$I = I_G\left(1 + \frac{I_s}{I_G}\right) = I_G\left(1 + \frac{R_G}{R_s}\right)$$

$$= I_G\left(\frac{R_s + R_G}{R_s}\right)$$

$$\boxed{R_s = \frac{I_G R_G}{I - I_G}}$$

Galvanometer to a Voltmeter

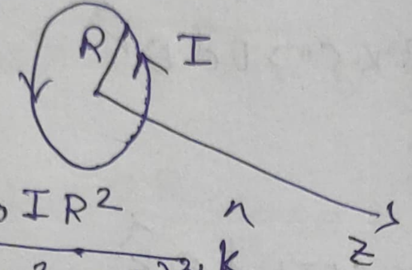




$R = R_p + R_{\omega} \simeq R_p$

For $R_p >> R_{\omega}$

Magnetic field due to a circular loop of current



$$\vec{B} = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}} \hat{k}$$

Along the axis

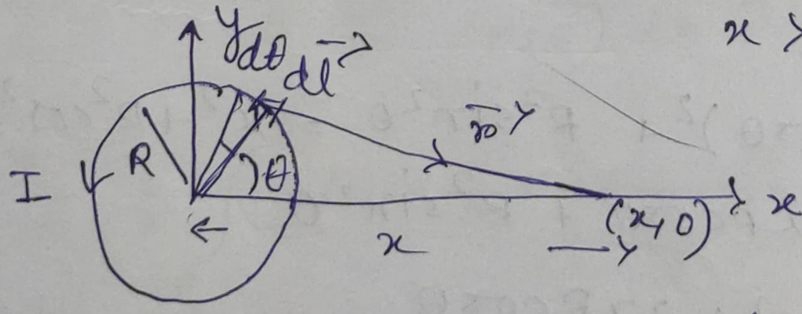
For $z >> R$

$$\vec{B} \simeq \frac{\mu_0 I R^2}{2z^3}\hat{\varepsilon}_r = \frac{\mu_0 \vec{m}}{2\pi z^3}$$

$|\vec{m}| = I\pi R^2$

Magnetic field in the plane of current loop

$x >> R$



$\vec{r} = (x - R\cos\theta)\hat{i} - R\sin\theta\hat{j}$

Coordinate at $\vec{dl}$ $= (R\cos\theta, R\sin\theta)$

$\vec{dl} = -dl\sin\theta\,\hat{i} + dl\cos\theta\,\hat{j}$, $dl = R\,d\theta$

$= -R\,d\theta\sin\theta\,\hat{i} + R\,d\theta\cos\theta\,\hat{j}$

$$\vec{dl} \times \vec{r} = (-R\sin\theta\, d\theta\, \hat{i} + R\cos\theta\, d\theta\, \hat{j})$$
$$\times \left[(x - R\cos\theta)\hat{i} - R\sin\theta\, \hat{j}\right]$$
$$= R^2 \sin^2\theta\, d\theta\, \hat{k} - R\cos\theta(x - R\cos\theta)\, d\theta\, \hat{k}$$
$$= R^2 \sin^2\theta\, d\theta\, \hat{k} - Rx\cos\theta\, d\theta\, \hat{k}$$
$$+ R^2\cos^2\theta\, d\theta\, \hat{k}$$
$$= R^2 d\theta\, \hat{k} - Rx\cos\theta\, d\theta\, \hat{k}$$
$$= \vec{dl} \times \vec{r}$$
$$\therefore r^3 = \left[(x - R\cos\theta)^2 + R^2\sin^2\theta\right]^{3/2}$$
$$\vec{B} = \frac{\mu_0 I}{4\pi} \oint \frac{\vec{dl} \times \vec{r}}{r^3}$$
$$= \frac{\mu_0 I R}{4\pi} \int_0^{2\pi} \frac{(R - x\cos\theta)\, d\theta}{\left[(x - R\cos\theta)^2 + R^2\sin^2\theta\right]^{3/2}}\, \hat{k}$$
$x >> R$
$$(x - R\cos\theta)^2 + R^2\sin^2\theta = x^2 + R^2\cos^2\theta$$
$$- 2xR\cos\theta + R^2\sin^2\theta$$
$$= x^2 + R^2 + 2xR\cos\theta$$
$$= x^2\left[1 - \frac{2R}{x}\cos\theta + \frac{R^2}{x^2}\right]$$

$$[(x-R\cos\theta)^2+R^2\sin^2\theta)]^{3/2}$$

$$= x^3\left[1-\frac{2R}{x}\cos\theta+\frac{R^2}{x^2}\right]^{3/2}$$

$$\frac{1}{[(x-R\cos\theta)^2+R^2\sin^2\theta]^{3/2}}$$

$$= \frac{1}{x^3}\left[1-\frac{2R}{x}\cos\theta+\frac{R^2}{x^2}\right]^{-3/2}$$

$$\simeq \frac{1}{x^3}\left[1+\frac{3R}{x}\cos\theta\right]$$

$$\vec{B} = \frac{\mu_0 IR}{4\pi}\int_0^{2\pi}(R-x\cos\theta)\frac{1}{x^3}\left(1+\frac{3R}{x}\cos\theta\right)$$

$$d\theta\,\hat{k}$$

$$= \frac{\mu_0 IR}{4\pi}\int_0^{2\pi}\Big(R-3R\cos^2-x\cos\theta$$

$$+\frac{3R^2}{x}\cos\theta\Big)\,d\theta\,\hat{k}$$

$$= \frac{\mu_0 IR^2}{4\pi x^3}\int_0^{2\pi}\left[1-3\left(\frac{1+\cos 2\theta}{2}\right)\right]d\theta\,\hat{k}$$

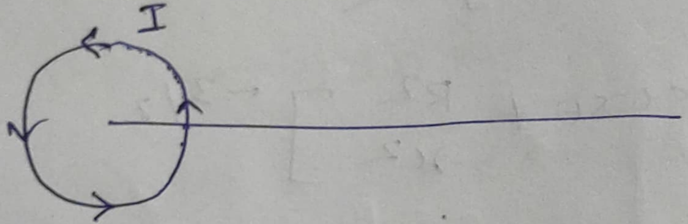
$$\cos 2\theta = 2\cos^2\theta-1$$

$$\vec{B} = \frac{\mu_0 IR^2}{4\pi x^3}\int_0^{2\pi}\left(-\frac{1}{2}\right)d\theta\,\hat{k}$$

$$\vec{B} = -\frac{\mu_0 I R^2}{4\pi \cdot x^3} \pi \hat{k}$$

$$\vec{m} = I \pi R^2 \hat{k}$$

$$\vec{B} = -\frac{\mu_0 \vec{m}}{4\pi x^3} \qquad x >> R$$



$\vec{m}$

$$\vec{B}\,(\text{along axis}) = \frac{\mu_0 \vec{m}}{2\pi z^3}$$

$z >> R$

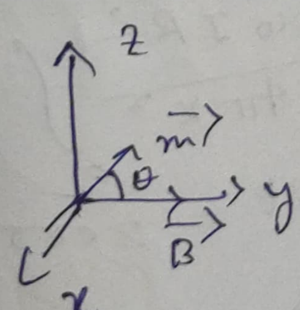
$\vec{B}$ (in the plane of loop)

$$= -\frac{\mu_0 \vec{m}}{4\pi x^3} \qquad x >> R$$

Potential energy of a magnetic dipole placed in a magnetic field:

$$\vec{B} = B\hat{j}$$

$$\vec{m} = m\cos\theta\,\hat{j} + m\sin\theta\,\hat{k}$$



Torque due to magnetic field $= \tau_{mag} = \vec{m} \times \vec{B}$

$$= (m\cos\theta\,\hat{j} + m\sin\theta\,\hat{k}) \times B\hat{j}$$

$= -mB\sin\theta\ \hat{i}$

External torque $\vec{\tau}_{ext} = mB\sin\theta\ \hat{i}$

change in potential energy in rotating the dipole from θ_1 to θ_2 is

$$U = \int_{\theta_1}^{\theta_2} \tau_{ext}\, d\theta$$

$$= mB\int_{\theta_1}^{\theta_2} \sin\theta\, d\theta$$

$$= -mB(\cos\theta_2 - \cos\theta_1)$$

Zero of potential energy $\theta_1 = \frac{\pi}{2}$

For an angle θ orientation

Potential energy $U = -mB$

$= -\vec{m}\cdot\vec{B}$

Minimum PE $= -mB$ for $\theta = 0$

Maximum PE $= +mB$ for $\theta = \pi$