This is larger than the value 2×10^{-7} Nm^{-1} quoted in the definition of the ampere. Hence it is important to eliminate the effect of the earth's magnetic field and other stray fields while standardising the ampere.

The direction of the force is downwards. This direction may be obtained by the directional property of cross product of vectors.

(b) When the current is flowing from south to north,

$\theta = 0°$

$f = 0$

Hence there is no force on the conductor.

Example 4.10

4.10 Torque on Current Loop, Magnetic Dipole

4.10.1 Torque on a rectangular current loop in a uniform magnetic field

We now show that a rectangular loop carrying a steady current I and placed in a uniform magnetic field experiences a torque. It does not experience a net force. This behaviour is analogous to that of electric dipole in a uniform electric field (Section 1.12).

We first consider the simple case when the rectangular loop is placed such that the uniform magnetic field **B** is in the plane of the loop. This is illustrated in Fig. 4.21(a).

The field exerts no force on the two arms AD and BC of the loop. It is perpendicular to the arm AB of the loop and exerts a force $\mathbf{F}_1$ on it which is directed into the plane of the loop. Its magnitude is,

$F_1 = I\,b\,B$

Similarly, it exerts a force $\mathbf{F}_2$ on the arm CD and $\mathbf{F}_2$ is directed out of the plane of the paper.

$F_2 = I\,b\,B = F_1$

Thus, the *net force* on the loop *is zero*. There is a torque on the loop due to the pair of forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Figure 4.21(b) shows a view of the loop from the AD end. It shows that the torque on the loop tends to rotate it anticlockwise. This torque is (in magnitude),

$$\tau = F_1\frac{a}{2} + F_2\frac{a}{2}$$

$$= IbB\frac{a}{2} + IbB\frac{a}{2} = I(ab)B$$

$$= I\,A\,B \qquad (4.26)$$

where $A = ab$ is the area of the rectangle.

We next consider the case when the plane of the loop, is not along the magnetic field, but makes an angle with it. We take the angle between the field and the normal to

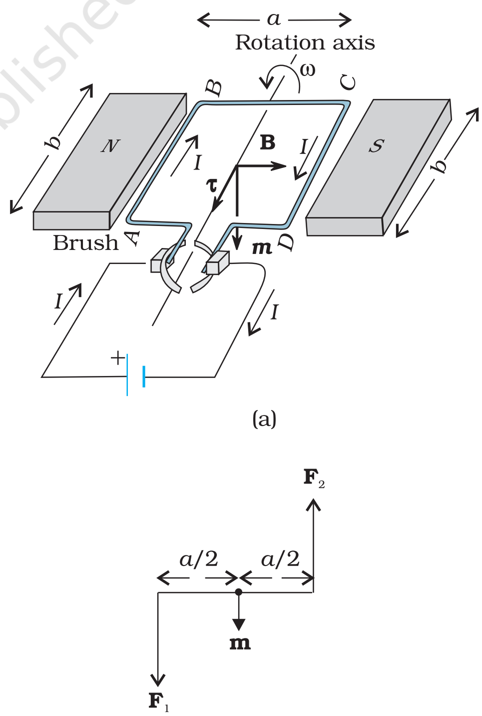
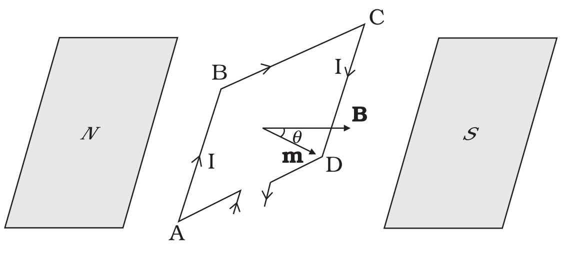
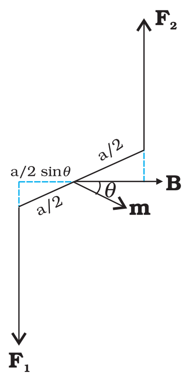


FIGURE 4.21 (a) A rectangular current-carrying coil in uniform magnetic field. The magnetic moment **m** points downwards. The torque **τ** is along the axis and tends to rotate the coil anticlockwise. (b) The couple acting on the coil.



(a)



(b)

FIGURE 4.22 (a) The area vector of the loop ABCD makes an arbitrary angle θ with the magnetic field. (b) Top view of the loop. The forces $\mathbf{F_1}$ and $\mathbf{F_2}$ acting on the arms AB and CD are indicated.

the coil to be angle θ (The previous case corresponds to $\theta = \pi/2$). Figure 4.22 illustrates this general case.

The forces on the arms BC and DA are equal, opposite, and act along the axis of the coil, which connects the centres of mass of BC and DA. Being collinear along the axis they cancel each other, resulting in no net force or torque. The forces on arms AB and CD are $\mathbf{F_1}$ and $\mathbf{F_2}$. They too are equal and opposite, with magnitude,

$$F_1 = F_2 = I\,b\,B$$

But they are not collinear! This results in a couple as before. The torque is, however, less than the earlier case when plane of loop was along the magnetic field. This is because the perpendicular distance between the forces of the couple has decreased. Figure 4.22(b) is a view of the arrangement from the AD end and it illustrates these two forces constituting a couple. The magnitude of the torque on the loop is,

$$\tau = F_1 \frac{a}{2}\sin\theta + F_2 \frac{a}{2}\sin\theta$$

$$= I\,ab\,B\sin\theta$$

$$= I\,A\,B\sin\theta \tag{4.27}$$

As $\theta \rightarrow 0$, the perpendicular distance between the forces of the couple also approaches zero. This makes the forces collinear and the net force and torque zero. The torques in Eqs. (4.26) and (4.27) can be expressed as vector product of the magnetic moment of the coil and the magnetic field. We define the *magnetic moment* of the current loop as,

$$\mathbf{m} = I\,\mathbf{A} \tag{4.28}$$

where the direction of the area vector **A** is given by the right-hand thumb rule and is directed into the plane of the paper in Fig. 4.21. Then as the angle between **m** and **B** is θ, Eqs. (4.26) and (4.27) can be expressed by one expression

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B} \tag{4.29}$$

This is analogous to the electrostatic case (Electric dipole of dipole moment $\mathbf{p}_e$ in an electric field **E**).

$$\boldsymbol{\tau} = \mathbf{p}_e \times \mathbf{E}$$

As is clear from Eq. (4.28), the dimensions of the magnetic moment are $[A][L^2]$ and its unit is Am^2.

From Eq. (4.29), we see that the torque $\boldsymbol{\tau}$ vanishes when **m** is either parallel or antiparallel to the magnetic field **B**. This indicates a state of equilibrium as there is no torque on the coil (this also applies to any object with a magnetic moment **m**). When **m** and **B** are parallel the equilibrium is a stable one. Any small rotation of the coil produces a torque which brings it back to its original position. When they are antiparallel, the equilibrium is unstable as any rotation produces a torque which increases with the amount of rotation. The presence of this torque is also the reason why a small magnet or any magnetic dipole aligns itself with the external magnetic field.

If the loop has N closely wound turns, the expression for torque, Eq. (4.29), still holds, with

$$\mathbf{m} = N I \mathbf{A} \tag{4.30}$$

Example 4.11 A 100 turn closely wound circular coil of radius 10 cm carries a current of 3.2 A. (a) What is the field at the centre of the coil? (b) What is the magnetic moment of this coil?

The coil is placed in a vertical plane and is free to rotate about a horizontal axis which coincides with its diameter. A uniform magnetic field of 2T in the horizontal direction exists such that initially the axis of the coil is in the direction of the field. The coil rotates through an angle of 90° under the influence of the magnetic field. (c) What are the magnitudes of the torques on the coil in the initial and final position? (d) What is the angular speed acquired by the coil when it has rotated by 90°? The moment of inertia of the coil is 0.1 kg m^2.

Solution

(a) From Eq. (4.16)

$$B = \frac{\mu_0 N I}{2R}$$

Here, N = 100; I = 3.2 A, and R = 0.1 m. Hence,

$$B = \frac{4\pi \times 10^{-7} \times 10^2 \times 3.2}{2 \times 10^{-1}} = \frac{4 \times 10^{-5} \times 10}{2 \times 10^{-1}} \quad (\text{using } \pi \times 3.2 = 10)$$

$$= 2 \times 10^{-3}\ \text{T}$$

The direction is given by the right-hand thumb rule.

(b) The magnetic moment is given by Eq. (4.30),

$m = N I A = N I \pi r^2 = 100 \times 3.2 \times 3.14 \times 10^{-2} = 10\ \text{A m}^2$

The direction is once again given by the right-hand thumb rule.

(c) $\tau = |\mathbf{m} \times \mathbf{B}|$ [from Eq. (4.29)]

$= m B \sin\theta$

Initially, θ = 0. Thus, initial torque τ_i = 0. Finally, $\theta = \pi/2$ (or 90º). Thus, final torque $\tau_f = m B = 10 \times 2 = 20$ N m.

(d) From Newton's second law,

$$\mathcal{I}\frac{d\omega}{dt} = mB\sin\theta$$

where $\mathcal{I}$ is the moment of inertia of the coil. From chain rule,

$$\frac{d\omega}{dt} = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \frac{d\omega}{d\theta}\omega$$

Using this,

$$\mathcal{I}\,\omega\, d\omega = mB\sin\theta\, d\theta$$

EXAMPLE 4.11

EXAMPLE 4.11

Integrating from $\theta = 0$ to $\theta = \pi/2$,

$$\mathcal{I} \int_0^{\omega_f} \omega \, d\omega = mB \int_0^{\pi/2} \sin\theta \, d\theta$$

$$\mathcal{I} \frac{\omega_f^2}{2} = -mB\cos\theta \Big|_0^{\pi/2} = mB$$

$$\omega_f = \left(\frac{2mB}{\mathcal{I}}\right)^{1/2} = \left(\frac{2\times 20}{10^{-1}}\right)^{1/2} = 20\text{s}^{-1}$$

EXAMPLE 4.12

Example 4.12

(a) A current-carrying circular loop lies on a smooth horizontal plane. Can a uniform magnetic field be set up in such a manner that the loop turns around itself (i.e., turns about the vertical axis).

(b) A current-carrying circular loop is located in a uniform external magnetic field. If the loop is free to turn, what is its orientation of stable equilibrium? Show that in this orientation, the flux of the total field (external field + field produced by the loop) is maximum.

(c) A loop of irregular shape carrying current is located in an external magnetic field. If the wire is flexible, why does it change to a circular shape?

Solution

(a) No, because that would require $\boldsymbol{\tau}$ to be in the vertical direction. But $\boldsymbol{\tau} = I\,\mathbf{A} \times \mathbf{B}$, and since $\mathbf{A}$ of the horizontal loop is in the vertical direction, τ would be in the plane of the loop for any $\mathbf{B}$.

(b) Orientation of stable equilibrium is one where the area vector $\mathbf{A}$ of the loop is in the direction of external magnetic field. In this orientation, the magnetic field produced by the loop is in the same direction as external field, both normal to the plane of the loop, thus giving rise to maximum flux of the total field.

(c) It assumes circular shape with its plane normal to the field to maximise flux, since for a given perimeter, a circle encloses greater area than any other shape.

4.10.2 Circular current loop as a magnetic dipole

In this section, we shall consider the elementary magnetic element: the current loop. We shall show that the magnetic field (at large distances) due to current in a circular current loop is very similar in behaviour to the electric field of an electric dipole. In Section 4.6, we have evaluated the magnetic field on the axis of a circular loop, of a radius R, carrying a steady current I. The magnitude of this field is [(Eq. (4.15)],

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

and its direction is along the axis and given by the right-hand thumb rule (Fig. 4.12). Here, x is the distance along the axis from the centre of the loop. For $x >> R$, we may drop the R^2 term in the denominator. Thus,

$$B = \frac{\mu_0 I R^2}{2x^3}$$

Note that the area of the loop $A = \pi R^2$. Thus,

$$B = \frac{\mu_0 I A}{2\pi x^3}$$

As earlier, we define the magnetic moment **m** to have a magnitude *IA*, $\mathbf{m} = I\,\mathbf{A}$. Hence,

$$B \simeq \frac{\mu_0 m}{2\pi x^3}$$

$$= \frac{\mu_0}{4\pi}\frac{2\mathbf{m}}{x^3} \qquad [4.31(a)]$$

The expression of Eq. [4.31(a)] is very similar to an expression obtained earlier for the electric field of a dipole. The similarity may be seen if we substitute,

$\mu_0 \rightarrow 1/\varepsilon_0$

$\mathbf{m} \rightarrow \mathbf{p}_e$ (electrostatic dipole)

$\mathbf{B} \rightarrow \mathbf{E}$ (electrostatic field)

We then obtain,

$$\mathbf{E} = \frac{2\mathbf{p}_e}{4\pi\varepsilon_0 x^3}$$

which is precisely the field for an electric dipole at a point on its axis. considered in Chapter 1, Section 1.10 [Eq. (1.20)].

It can be shown that the above analogy can be carried further. We had found in Chapter 1 that the electric field on the perpendicular bisector of the dipole is given by [See Eq.(1.21)],

$$E \simeq \frac{\mathbf{p}_e}{4\pi\varepsilon_0 x^3}$$

where x is the distance from the dipole. If we replace $\mathbf{p} \rightarrow \mathbf{m}$ and $\mu_0 \rightarrow 1/\varepsilon_0$ in the above expression, we obtain the result for **B** for a point *in the plane of the loop* at a distance x from the centre. For $x >> R$,

$$\mathbf{B} \simeq \frac{\mu_0}{4\pi}\frac{\mathbf{m}}{x^3}; \qquad x >> R \qquad [4.31(b)]$$

The results given by Eqs. [4.31(a)] and [4.31(b)] become exact for a *point* magnetic dipole.

The results obtained above can be shown to apply to any planar loop: a planar current loop is equivalent to a magnetic dipole of dipole moment $\mathbf{m} = I\,\mathbf{A}$, which is the analogue of electric dipole moment **p**. Note, however, a fundamental difference: an electric dipole is built up of two elementary units — the charges (or electric monopoles). In magnetism, a magnetic dipole (or a current loop) is the most elementary element. The equivalent of electric charges, i.e., magnetic monopoles, are not known to exist.

We have shown that a current loop (i) produces a magnetic field (see Fig. 4.12) and behaves like a magnetic dipole at large distances, and

(ii) is subject to torque like a magnetic needle. This led Ampere to suggest that all magnetism is due to circulating currents. This seems to be partly true and no magnetic monopoles have been seen so far. However, elementary particles such as an electron or a proton also carry an *intrinsic* magnetic moment, not accounted by circulating currents.

4.10.3 The magnetic dipole moment of a revolving electron

In Chapter 12 we shall read about the Bohr model of the hydrogen atom. You may perhaps have heard of this model which was proposed by the Danish physicist Niels Bohr in 1911 and was a stepping stone to a new kind of mechanics, namely, quantum mechanics. In the Bohr model, the electron (a negatively charged particle) revolves around a positively charged nucleus much as a planet revolves around the sun. The force in the former case is electrostatic (Coulomb force) while it is gravitational for the planet-Sun case. We show this Bohr picture of the electron in Fig. 4.23.

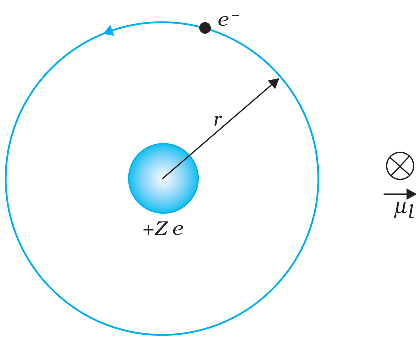


FIGURE 4.23 In the Bohr model of hydrogen-like atoms, the negatively charged electron is revolving with uniform speed around a centrally placed positively charged (+*Z e*) nucleus. The uniform circular motion of the electron constitutes a current. The direction of the magnetic moment is into the plane of the paper and is indicated separately by ⊗.

The electron of charge (–*e*) ($e = +\ 1.6 \times 10^{-19}$ C) performs uniform circular motion around a stationary heavy nucleus of charge +*Ze*. This constitutes a current *I*, where,

$$I = \frac{e}{T} \tag{4.32}$$

and *T* is the time period of revolution. Let *r* be the orbital radius of the electron, and *v* the orbital speed. Then,

$$T = \frac{2\pi r}{v} \tag{4.33}$$

Substituting in Eq. (4.32), we have $I = ev/2\pi r$.

There will be a magnetic moment, usually denoted by μ_l, associated with this circulating current. From Eq. (4.28) its magnitude is, $\mu_l = I\pi r^2 = evr/2$.

The direction of this magnetic moment is into the plane of the paper in Fig. 4.23. [This follows from the right-hand rule discussed earlier and the fact that the negatively charged electron is moving anticlockwise, leading to a clockwise current.] Multiplying and dividing the right-hand side of the above expression by the electron mass m_e, we have,

$$\mu_l = \frac{e}{2m_e}(m_e v r)$$

$$= \frac{e}{2m_e} l \tag{4.34(a)}$$

Here, *l* is the magnitude of the angular momentum of the electron about the central nucleus ("orbital" angular momentum). Vectorially,

$$\boldsymbol{\mu}_l = -\frac{e}{2m_e}\boldsymbol{l} \tag{4.34(b)}$$

The negative sign indicates that the angular momentum of the electron is opposite in direction to the magnetic moment. Instead of electron with charge (– e), if we had taken a particle with charge (+q), the angular momentum and magnetic moment would be in the same direction. The ratio

$$\frac{\mu_l}{l} = \frac{e}{2m_e} \qquad (4.35)$$

is called the *gyromagnetic ratio* and is a constant. Its value is 8.8×10^{10} C /kg for an electron, which has been verified by experiments.

The fact that even at an atomic level there is a magnetic moment, confirms Ampere's bold hypothesis of atomic magnetic moments. This according to Ampere, would help one to explain the magnetic properties of materials. Can one assign a value to this atomic dipole moment? The answer is Yes. One can do so within the Bohr model. Bohr hypothesised that the angular momentum assumes a discrete set of values, namely,

$$l = \frac{nh}{2\pi} \qquad (4.36)$$

where n is a natural number, n = 1, 2, 3, ... and h is a constant named after Max Planck (Planck's constant) with a value $h = 6.626 \times 10^{-34}$ J s. This condition of discreteness is called the *Bohr quantisation condition*. We shall discuss it in detail in Chapter 12. Our aim here is merely to use it to calculate the elementary dipole moment. Take the value n = 1, we have from Eq. (4.34) that,

$$(\mu_l)_{\min} = \frac{e}{4\pi m_e} h$$

$$= \frac{1.60 \times 10^{-19} \times 6.63 \times 10^{-34}}{4 \times 3.14 \times 9.11 \times 10^{-31}}$$

$$= 9.27 \times 10^{-24} \text{ Am}^2 \qquad (4.37)$$

where the subscript 'min' stands for minimum. This value is called the *Bohr magneton*.

Any charge in uniform circular motion would have an associated magnetic moment given by an expression similar to Eq. (4.34). This dipole moment is labelled as the *orbital magnetic moment*. Hence, the subscript 'l' in μ_l. Besides the orbital moment, the electron has an *intrinsic* magnetic moment, which has the same numerical value as given in Eq. (4.37). It is called the *spin magnetic moment*. But we hasten to add that it is not as though the electron is spinning. The electron is an elementary particle and it does not have an axis to spin around like a top or our earth. Nevertheless, it does possess this *intrinsic* magnetic moment. The microscopic roots of magnetism in iron and other materials can be traced back to this intrinsic spin magnetic moment.

4.11 The Moving Coil Galvanometer

Currents and voltages in circuits have been discussed extensively in Chapters 3. But how do we measure them? How do we claim that current in a circuit is 1.5 A or the voltage drop across a resistor is 1.2 V? Figure 4.24 exhibits a very useful instrument for this purpose: the *moving*

PHYSICS

Conversion of galvanometer into ammeter and voltmeter:
www.citycollegiate.com/galvanometer_XIIa.htm

coil galvanometer (MCG). It is a device whose principle can be understood on the basis of our discussion in Section 4.10.

The galvanometer consists of a coil, with many turns, free to rotate about a fixed axis (Fig. 4.24), in a uniform radial magnetic field. There is a cylindrical soft iron core which not only makes the field radial but also increases the strength of the magnetic field. When a current flows through the coil, a torque acts on it. This torque is given by Eq. (4.26) to be

$$\tau = NI\,AB$$

where the symbols have their usual meaning. Since the field is radial by design, we have taken sin $\theta = 1$ in the above expression for the torque. The magnetic torque *NIAB* tends to rotate the coil. A spring S_p provides a counter torque $k\phi$ that balances the magnetic torque *NIAB;* resulting in a steady angular deflection ϕ. In equilibrium

$$k\phi = NI\,AB$$

where k is the torsional constant of the spring; i.e. the restoring torque per unit twist. The deflection ϕ is indicated on the scale by a pointer attached to the spring. We have

$$\phi = \left(\frac{NAB}{k}\right) I \qquad (4.38)$$

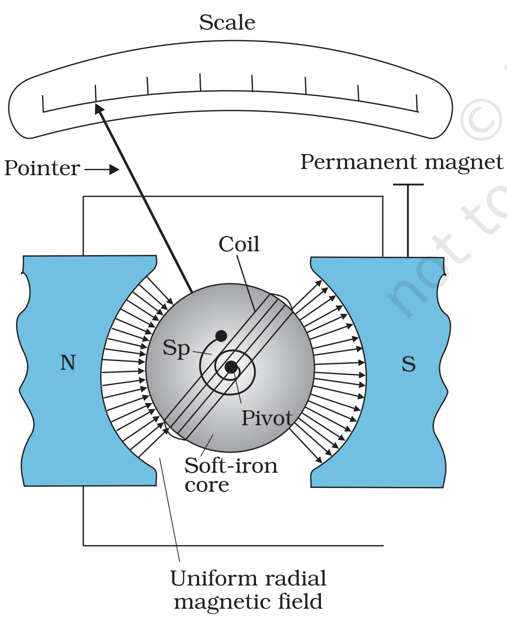


FIGURE 4.24 The moving coil galvanometer. Its elements are described in the text. Depending on the requirement, this device can be used as a current detector or for measuring the value of the current (ammeter) or voltage (voltmeter).

The quantity in brackets is a constant for a given galvanometer.

The galvanometer can be used in a number of ways. It can be used as a detector to check if a current is flowing in the circuit. We have come across this usage in the Wheatstone's bridge arrangement. In this usage the neutral position of the pointer (when no current is flowing through the galvanometer) is in the middle of the scale and not at the left end as shown in Fig.4.24. Depending on the direction of the current, the pointer's deflection is either to the right or the left.

The galvanometer cannot as such be used as an ammeter to measure the value of the current in a given circuit. This is for two reasons: (i) Galvanometer is a very sensitive device, it gives a full-scale deflection for a current of the order of μA. (ii) For measuring currents, the galvanometer has to be connected in series, and as it has a large resistance, this will change the value of the current in the circuit. To overcome these difficulties, one attaches a small resistance r_s, called *shunt resistance*, in parallel with the galvanometer coil; so that most of the current passes through the shunt. The resistance of this arrangement is,

$$R_G\, r_s / (R_G + r_s) \simeq r_s \qquad \text{if} \quad R_G >> r_s$$

If r_s has small value, in relation to the resistance of the rest of the circuit R_c, the effect of introducing the measuring instrument is also small and negligible. This arrangement is schematically shown in Fig. 4.25. The scale of this ammeter is calibrated and then graduated to read off the current value with ease. We define the *current sensitivity of the galvanometer as the deflection per unit current*. From Eq. (4.38) this current sensitivity is,

$$\frac{\phi}{I} = \frac{NAB}{k} \tag{4.39}$$

A convenient way for the manufacturer to increase the sensitivity is to increase the number of turns *N*. We choose galvanometers having sensitivities of value, required by our experiment.

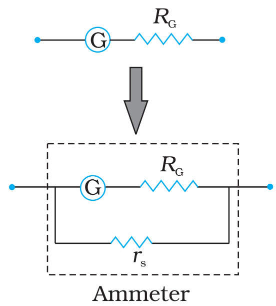


FIGURE 4.25 Conversion of a galvanometer (G) to an ammeter by the introduction of a shunt resistance r_s of very small value in parallel.

The galvanometer can also be used as a voltmeter to measure the voltage across a given section of the circuit. For this it must be connected *in parallel* with that section of the circuit. Further, it must draw a very small current, otherwise the voltage measurement will disturb the original set up by an amount which is very large. Usually we like to keep the disturbance due to the measuring device below one per cent. To ensure this, a large resistance *R* is connected *in series* with the galvanometer. This arrangement is schematically depicted in Fig.4.26. Note that the resistance of the voltmeter is now,

$R_G + R \simeq R$: large

The scale of the voltmeter is calibrated to read off the voltage value with ease. We define the *voltage sensitivity as the deflection per unit voltage*. From Eq. (4.38),

$$\frac{\phi}{V} = \left(\frac{NAB}{k}\right)\frac{I}{V} = \left(\frac{NAB}{k}\right)\frac{1}{R} \tag{4.40}$$

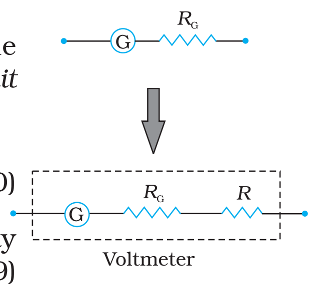


FIGURE 4.26 Conversion of a galvanometer (G) to a voltmeter by the introduction of a resistance *R* of large value in series.

An interesting point to note is that increasing the current sensitivity may not necessarily increase the voltage sensitivity. Let us take Eq. (4.39) which provides a measure of current sensitivity. If $N \rightarrow 2N$, i.e., we double the number of turns, then

$$\frac{\phi}{I} \rightarrow 2\frac{\phi}{I}$$

Thus, the current sensitivity doubles. However, the resistance of the galvanometer is also likely to double, since it is proportional to the length of the wire. In Eq. (4.40), $N \rightarrow 2N$, and $R \rightarrow 2R$, thus the voltage sensitivity,

$$\frac{\phi}{V} \rightarrow \frac{\phi}{V}$$

remains unchanged. So in general, the modification needed for conversion of a galvanometer to an ammeter will be different from what is needed for converting it into a voltmeter.

Example 4.13 In the circuit (Fig. 4.27) the current is to be measured. What is the value of the current if the ammeter shown (a) is a galvanometer with a resistance R_G = 60.00 Ω; (b) is a galvanometer described in (a) but converted to an ammeter by a shunt resistance r_s = 0.02 Ω; (c) is an ideal ammeter with zero resistance?

EXAMPLE 4.13

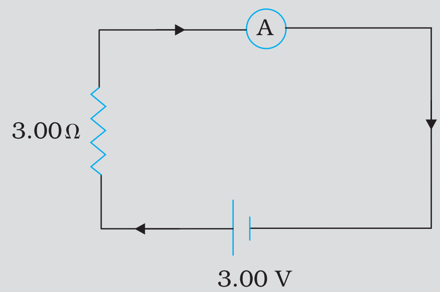


FIGURE 4.27

Solution

(a) Total resistance in the circuit is,

$R_G + 3 = 63\,\Omega$. Hence, I = 3/63 = 0.048 A.

(b) Resistance of the galvanometer converted to an ammeter is,

$$\frac{R_G\, r_s}{R_G + r_s} = \frac{60\ \ \Omega \times 0.02\Omega}{(60 + 0.02)\Omega} \simeq 0.02\Omega$$

Total resistance in the circuit is,

$0.02\,\Omega + 3\,\Omega = 3.02\,\Omega$. Hence, I = 3/3.02 = 0.99 A.

(c) For the ideal ammeter with zero resistance,

I = 3/3 = 1.00 A

EXAMPLE 4.13

SUMMARY

1. The total force on a charge q moving with velocity **v** in the presence of magnetic and electric fields **B** and **E,** respectively is called the *Lorentz force*. It is given by the expression:
   $\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B} + \mathbf{E})$
   The magnetic force $q\,(\mathbf{v} \times \mathbf{B})$ is normal to **v** and work done by it is zero.
2. A straight conductor of length l and carrying a steady current I experiences a force **F** in a uniform external magnetic field **B**,
   $\mathbf{F} = I\,\mathbf{l} \times \mathbf{B}$
   where $|\mathbf{l}| = l$ and the direction of **l** is given by the direction of the current.
3. In a uniform magnetic field **B**, a charge q executes a circular orbit in a plane normal to **B**. Its frequency of uniform circular motion is called the *cyclotron frequency* and is given by:
   $$\nu_c = \frac{qB}{2\pi m}$$
   This frequency is independent of the particle's speed and radius. This fact is exploited in a machine, the cyclotron, which is used to accelerate charged particles.
4. The *Biot-Savart* law asserts that the magnetic field d**B** due to an element d**l** carrying a steady current I at a point P at a distance r from the current element is:
   $$d\mathbf{B} = \frac{\mu_0}{4\pi} I \frac{d\mathbf{l} \times \mathbf{r}}{r^3}$$

To obtain the total field at P, we must integrate this vector expression over the entire length of the conductor.

5. The magnitude of the magnetic field due to a circular coil of radius R carrying a current I at an axial distance x from the centre is

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

At the centre this reduces to

$$B = \frac{\mu_0 I}{2R}$$

6. *Ampere's Circuital Law:* Let an open surface S be bounded by a loop C. Then the Ampere's law states that $\oint_C \mathbf{B}.d\mathbf{l} = \mu_0 I$ where I refers to the current passing through S. The sign of I is determined from the right-hand rule. We have discussed a simplified form of this law. If **B** is directed along the tangent to every point on the perimeter L of a closed curve and is constant in magnitude along perimeter then,

$BL = \mu_0 I_e$

where I_e is the net current enclosed by the closed circuit.

7. The magnitude of the magnetic field at a distance R from a long, straight wire carrying a current I is given by:

$$B = \frac{\mu_0 I}{2\pi R}$$

The field lines are circles concentric with the wire.

8. The magnitude of the field B inside a *long solenoid* carrying a current I is

$B = \mu_0 n I$

where n is the number of turns per unit length. For a *toroid* one obtains,

$$B = \frac{\mu_0 N I}{2\pi r}$$

where N is the total number of turns and r is the average radius.

9. Parallel currents attract and anti-parallel currents repel.

10. A planar loop carrying a current I, having N closely wound turns, and an area A possesses a magnetic moment **m** where,

m = N I **A**

and the direction of **m** is given by the right-hand thumb rule : curl the palm of your right hand along the loop with the fingers pointing in the direction of the current. The thumb sticking out gives the direction of **m** (and **A**)

When this loop is placed in a uniform magnetic field **B**, the force **F** on it is: $F = 0$

And the torque on it is,

$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$

In a moving coil galvanometer, this torque is balanced by a counter-torque due to a spring, yielding

$k\phi = NI\,AB$

where ϕ is the equilibrium deflection and k the torsion constant of the spring.

11. An electron moving around the central nucleus has a magnetic moment μ_l given by:

$$\mu_l = \frac{e}{2m} l$$

where l is the magnitude of the angular momentum of the circulating electron about the central nucleus. The smallest value of μ_l is called the Bohr magneton μ_B and it is $\mu_B = 9.27 \times 10^{-24}$ J/T

12. A moving coil galvanometer can be converted into a ammeter by introducing a shunt resistance r_s, of small value in parallel. It can be converted into a voltmeter by introducing a resistance of a large value in series.

Physical Quantity	Symbol	Nature	Dimensions	Units	Remarks
Permeability of free space	μ_0	Scalar	$[MLT^{-2}A^{-2}]$	$T\ m\ A^{-1}$	$4\pi \times 10^{-7}\ T\ m\ A^{-1}$
Magnetic Field	**B**	Vector	$[M\ T^{-2}A^{-1}]$	T (telsa)	
Magnetic Moment	**m**	Vector	$[L^2A]$	$A\ m^2$ or J/T	
Torsion Constant	k	Scalar	$[M\ L^2T^{-2}]$	$N\ m\ rad^{-1}$	Appears in MCG

POINTS TO PONDER

1. Electrostatic field lines originate at a positive charge and terminate at a negative charge or fade at infinity. Magnetic field lines always form closed loops.
2. The discussion in this Chapter holds only for steady currents which do not vary with time.
   When currents vary with time Newton's third law is valid only if momentum carried by the electromagnetic field is taken into account.
3. Recall the expression for the Lorentz force,

   $$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B} + \mathbf{E})$$

   This velocity dependent force has occupied the attention of some of the greatest scientific thinkers. If one switches to a frame with instantaneous velocity **v**, the magnetic part of the force vanishes. The motion of the charged particle is then explained by arguing that there exists an appropriate electric field in the new frame. We shall not discuss the details of this mechanism. However, we stress that the resolution of this paradox implies that electricity and magnetism are linked phenomena (*electromagnetism*) and that the Lorentz force expression *does not* imply a universal preferred frame of reference in nature.
4. Ampere's Circuital law is not independent of the Biot-Savart law. It can be derived from the Biot-Savart law. Its relationship to the Biot-Savart law is similar to the relationship between Gauss's law and Coulomb's law.

EXERCISES

4.1 A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field **B** at the centre of the coil?

4.2 A long straight wire carries a current of 35 A. What is the magnitude of the field **B** at a point 20 cm from the wire?

4.3 A long straight wire in the horizontal plane carries a current of 50 A in north to south direction. Give the magnitude and direction of **B** at a point 2.5 m east of the wire.

4.4 A horizontal overhead power line carries a current of 90 A in east to west direction. What is the magnitude and direction of the magnetic field due to the current 1.5 m below the line?

4.5 What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30º with the direction of a uniform magnetic field of 0.15 T?

4.6 A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire?

4.7 Two long and parallel straight wires A and B carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A.

4.8 A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of **B** inside the solenoid near its centre.

4.9 A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil makes an angle of 30º with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil?

4.10 Two moving coil meters, M_1 and M_2 have the following particulars:

$R_1 = 10\ \Omega$, $N_1 = 30$,

$A_1 = 3.6 \times 10^{-3}\ \text{m}^2$, $B_1 = 0.25$ T

$R_2 = 14\ \Omega$, $N_2 = 42$,

$A_2 = 1.8 \times 10^{-3}\ \text{m}^2$, $B_2 = 0.50$ T

(The spring constants are identical for the two meters).

Determine the ratio of (a) current sensitivity and (b) voltage sensitivity of M_2 and M_1.

4.11 In a chamber, a uniform magnetic field of 6.5 G (1 G = 10^{-4} T) is maintained. An electron is shot into the field with a speed of $4.8 \times 10^6\ \text{m s}^{-1}$ normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. ($e = 1.5 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg)

4.12 In Exercise 4.11 obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron? Explain.

4.13 (a) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of 60°

with the normal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.

(b) Would your answer change, if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are also unaltered.)

ADDITIONAL EXERCISES

4.14 Two concentric circular coils X and Y of radii 16 cm and 10 cm, respectively, lie in the same vertical plane containing the north to south direction. Coil X has 20 turns and carries a current of 16 A; coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise, and clockwise in Y, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre.

4.15 A magnetic field of 100 G (1 G = 10^{-4} T) is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about 10^{-3} m^2. The maximum current-carrying capacity of a given coil of wire is 15 A and the number of turns per unit length that can be wound round a core is at most 1000 turns m^{-1}. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not ferromagnetic.

4.16 For a circular coil of radius *R* and *N* turns carrying current *I*, the magnitude of the magnetic field at a point on its axis at a distance *x* from its centre is given by,

$$B = \frac{\mu_0 I R^2 N}{2\left(x^2 + R^2\right)^{3/2}}$$

(a) Show that this reduces to the familiar result for field at the centre of the coil.

(b) Consider two parallel co-axial circular coils of equal radius *R*, and number of turns *N*, carrying equal currents in the same direction, and separated by a distance *R*. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small as compared to *R*, and is given by,

$$B = 0.72\frac{\mu_0 N I}{R}, \text{ approximately.}$$

[Such an arrangement to produce a nearly uniform magnetic field over a small region is known as *Helmholtz coils*.]

4.17 A toroid has a core (non-ferromagnetic) of inner radius 25 cm and outer radius 26 cm, around which 3500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field (a) outside the toroid, (b) inside the core of the toroid, and (c) in the empty space surrounded by the toroid.

4.18 Answer the following questions:

(a) A magnetic field that varies in magnitude from point to point but has a constant direction (east to west) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle?

(b) A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction, and comes out of it following a complicated trajectory. Would its final speed equal the initial speed if it suffered no collisions with the environment?

(c) An electron travelling west to east enters a chamber having a uniform electrostatic field in north to south direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron from deflecting from its straight line path.

4.19 An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV, enters a region with uniform magnetic field of 0.15 T. Determine the trajectory of the electron if the field (a) is transverse to its initial velocity, (b) makes an angle of 30º with the initial velocity.

4.20 A magnetic field set up using Helmholtz coils (described in Exercise 4.16) is uniform in a small region and has a magnitude of 0.75 T. In the same region, a uniform electrostatic field is maintained in a direction normal to the common axis of the coils. A narrow beam of (single species) charged particles all accelerated through 15 kV enters this region in a direction perpendicular to both the axis of the coils and the electrostatic field. If the beam remains undeflected when the electrostatic field is 9.0×10^{-5} V m^{-1}, make a simple guess as to what the beam contains. Why is the answer not unique?

4.21 A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires.

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero?

(b) What will be the total tension in the wires if the direction of current is reversed keeping the magnetic field same as before? (Ignore the mass of the wires.) $g = 9.8$ m s^{-2}.

4.22 The wires which connect the battery of an automobile to its starting motor carry a current of 300 A (for a short time). What is the force per unit length between the wires if they are 70 cm long and 1.5 cm apart? Is the force attractive or repulsive?

4.23 A uniform magnetic field of 1.5 T exists in a cylindrical region of radius10.0 cm, its direction parallel to the axis along east to west. A wire carrying current of 7.0 A in the north to south direction passes through this region. What is the magnitude and direction of the force on the wire if,

(a) the wire intersects the axis,

(b) the wire is turned from N-S to northeast-northwest direction,

(c) the wire in the N-S direction is lowered from the axis by a distance of 6.0 cm?

4.24 A uniform magnetic field of 3000 G is established along the positive *z*-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. What is the torque on the loop in the different cases shown in Fig. 4.28? What is the force on each case? Which case corresponds to stable equilibrium?

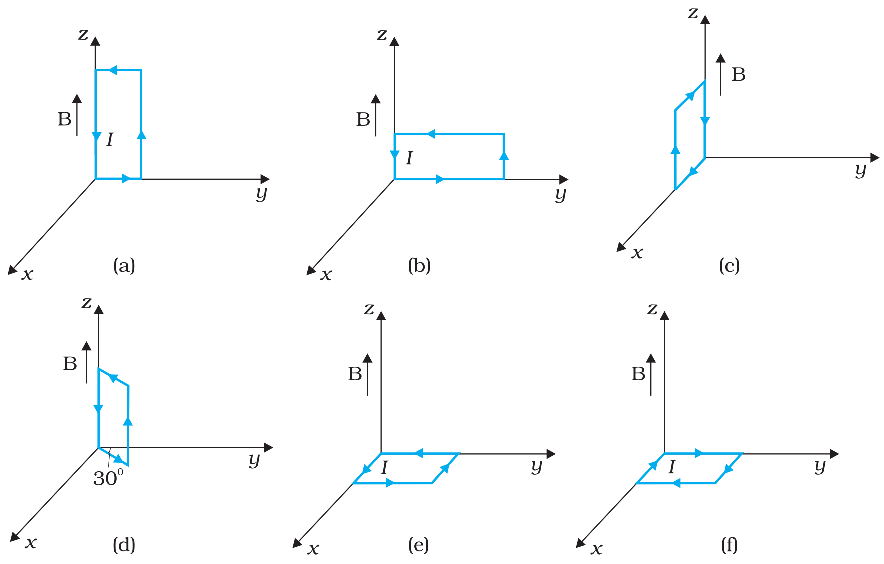


FIGURE 4.28

4.25 A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the

(a) total torque on the coil,

(b) total force on the coil,

(c) average force on each electron in the coil due to the magnetic field?

(The coil is made of copper wire of cross-sectional area 10^{-5} m^2, and the free electron density in copper is given to be about 10^{29} m^{-3}.)

4.26 A solenoid 60 cm long and of radius 4.0 cm has 3 layers of windings of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid (near its centre) normal to its axis; both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire? g = 9.8 m s^{-2}.

4.27 A galvanometer coil has a resistance of 12 Ω and the metre shows full scale deflection for a current of 3 mA. How will you convert the metre into a voltmeter of range 0 to 18 V?

4.28 A galvanometer coil has a resistance of 15 Ω and the metre shows full scale deflection for a current of 4 mA. How will you convert the metre into an ammeter of range 0 to 6 A?