Question 3: Evaluate the Taylor Series for $f(x) = x^3 - 10x^2 + 6$ at $x = 3$.

Solution: First, we will find the derivatives of the given function.

$f(x) = x^3 - 10x^2 + 6 \Rightarrow f(3) = -57$

$f'(x) = 3x^2 - 20x \Rightarrow f'(3) = 33$

$f''(x) = 6x - 20 \Rightarrow f''(3) = -2$

$f'''(x) = 6 \Rightarrow f'''(3) = 6$

$f''''(x) = 0$

Therefore, the required series is:

$$x^3 - 10x^2 + 6 = \sum_{n=0}^{\infty} \frac{f^{(n)}(3)}{n!}(x-3)^n$$

$$= f(3) + f'(3)(x-3) + \frac{f''(3)}{2!}(x-3)^2 + \frac{f'''(3)}{3!}(x-3)^3 + 0$$

$$= -57 - 33(x-3) - (x-3)^2 + (x-3)^3$$

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