

Question 2: Evaluate the Taylor Series for $f(x) = \cos(x)$ for $x = 0$.

Solution: We need to take the derivatives of the $\cos x$ and evaluate them at $x = 0$.

$$f(x) = \cos x \Rightarrow f(0) = 1$$

$$f'(x) = -\sin x \Rightarrow f'(0) = 0$$

$$f''(x) = -\cos x \Rightarrow f''(0) = -1$$

$$f'''(x) = \sin x \Rightarrow f'''(0) = 0$$

$$f^{(4)}(x) = \cos x \Rightarrow f^{(4)}(0) = 1$$

$$f^{(5)}(x) = -\sin x \Rightarrow f^{(5)}(0) = 0$$

$$f^{(6)}(x) = -\cos x \Rightarrow f^{(6)}(0) = -1$$

Therefore, according to the Taylor series expansion;

$$\begin{aligned}\cos x &= \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \\ &= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \frac{f^{(5)}(0)}{5!}x^5 + \dots \\ &= 1 + 0 - \frac{1}{2!}x^2 + 0 + \frac{1}{4!}x^4 + 0 - \frac{1}{6!}x^6 + \dots\end{aligned}$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$