

Problems and Solutions

Question 1: Determine the Taylor series at $x=0$ for $f(x) = e^x$

Solution: Given: $f(x) = e^x$

Differentiate the given equation,

$$f'(x) = e^x$$

$$f''(x) = e^x$$

$$f'''(x) = e^x$$

At $x=0$, we get

$$f'(0) = e^0 = 1$$

$$f''(0) = e^0 = 1$$

$$f'''(0) = e^0 = 1$$

When Taylor series at $x=0$, then the Maclaurin series is

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

$$e^x = 1 + x(1) + (x^2/2!)(1) + (x^3/3!)(1) + \dots$$

Therefore, $e^x = 1 + x + (x^2/2!) + (x^3/3!) + \dots$