speedometer

* Avg. speed is always +ve in contrast to avg. velocity which being a vector can be +ve or -ve.

* $\underline{avg.}$ x $\underline{speed}$ x & $\underline{avg.}$ x $\underline{velocity}$

⇒ $\dfrac{\text{distance travelled}}{\text{time taken}}$

* scaler quantity.

<u>Ques.</u>

a particle travels with an avg. speed of 40 km/h for half of the journey distance & with avg speed 60 km/h. for the remaining half. calculate the avg speed of the particle for its entire journey.

⇒ $\left(\dfrac{x}{2}\right)$ ⇒ $d = v \times t$.

⇒ $t = \dfrac{x.}{2 \times 40.}$ $\dfrac{x}{80}$

⇒ ⇒ $t = \dfrac{x}{120}$

⇒ $\dfrac{x}{80} + \dfrac{x}{120}$

$\dfrac{3x + 2x}{240}$ ⇒ $\dfrac{5x}{240} = t$

⇒ ⇒ $\dfrac{d}{t}$ ⇒ $\dfrac{x \cdot 240}{5x}$ ⇒ $\boxed{48}$

Position ⟹ the position of a particle refers to its location in the space at a certain moment of time. it is concerned with the question - "where is the particle at a partificular moment of time.

Ques. a particle travels total distance d, during its journey it travels an equal distances of d/n with average speed $v_1, v_2 --- v_n$. calculate the its avg. speed over entire journey.

Ans.

$$avg \ speed = \frac{D}{t_1 + t_2 ------ + t_n}$$

$$\Rightarrow \frac{D}{\frac{D/n}{v_1} + \frac{D/n}{v_2} + ---- + \frac{D_n/n}{v_n}}$$

Ques. a particle travels with an avg. speed of 40 km/h for half of total journey time, and with avg. speed of 60 km/h for remaing half time. calculate avg, speed.

⟹
$$d = v \times t$$
$$d_1 = \frac{40t}{2}$$
$$\Rightarrow d_2 = \frac{60t}{2}$$

⟹ $d_1 + d_2 = \frac{so}{2} \cdot t$

$$d_1 + d_2 = 50t.$$

⟹ ⟹ so.

$$\boxed{avg. \ speed = \frac{V_1 + V_2}{2}}$$ when two velocity in half times

equal time $\Rightarrow$ $\dfrac{v_1 + v_2}{2}$ $\Rightarrow$ avg. speed.

Ques.

$$t_1 = \dfrac{d/2}{v_1}$$

$$\dfrac{D}{2} = \dfrac{v_2 t_2}{2} + \dfrac{v_3 t_2}{2}$$

$$\Rightarrow \quad \dfrac{D}{2} = v_1 t_1$$

$$\Rightarrow \quad t_2 = \dfrac{D}{(v_2 + v_3)}$$

$$\Rightarrow \quad t_1 + t_2 = \dfrac{D}{2v_1} + \dfrac{D}{v_2 + v_3}$$

$$\Rightarrow \quad \boxed{\dfrac{2v_1 (v_2 + v_3)}{(v_2 + v_3 + 2v_1)}}$$

Ques $\Rightarrow$. Birds.

$\Rightarrow$ (18 Km/s)

$\Rightarrow$ $\underline{\text{dist. by the bird.}}$

$\Rightarrow$ $\underline{0.5 \text{ km.}}$

$\Rightarrow \quad t = \dfrac{0.5}{72}$ $\qquad t = \dfrac{0.5}{54}$

* Displacement can be negative positive, or zero.
* If the motion of a particle is along a straight line 4
and in same direction then, avg velocity = avg. speed.

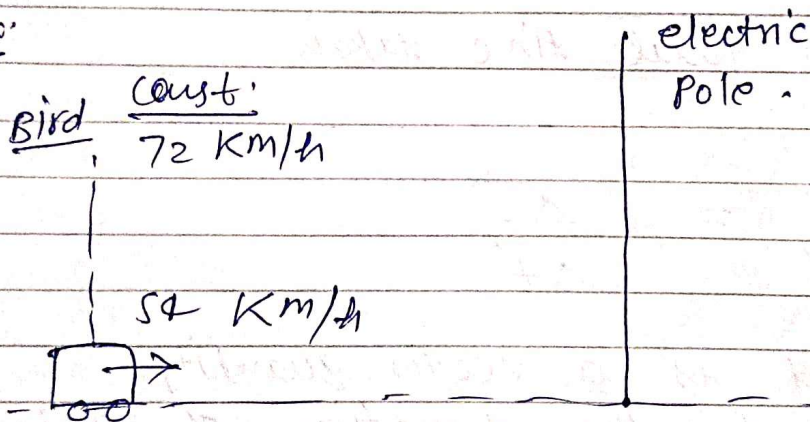$\Rightarrow$ (B) $\Rightarrow$ $\dfrac{0.5 \times 18}{54 \times 5}$ $\Rightarrow \dfrac{0.5}{15}$ $\dfrac{0.1}{3}$

$\dfrac{144 \cdot 3}{108}$

$t \Rightarrow \dfrac{1}{2 \times 72} \Rightarrow \boxed{\dfrac{1}{144}}$ $\Rightarrow$ $\boxed{\dfrac{1}{108}}$ $\overset{8}{\downarrow}$

Ques: Calculate no. of oscilation covered
by the birds in finish time.

$\Rightarrow$

Ques. ex.

| electric
| Pole.

Bird $\underrightarrow{\text{const}}$
72 Km/h

54 Km/h

$\Rightarrow$ ∞ (infinite oscilation).

$\Rightarrow$ total time. $\Rightarrow$ $\dfrac{0.50}{54}$ $\checkmark$

$\Rightarrow$ Note:
in this question birds complete ∞ oscilations
because when the birds remain very very
small distance. ∞ oscilation completed
very rapidly.

* the average velocity is a vector in the direction of displacement.
* avg. speed is, in general, greater than the magnitude of avg. velocity

* Note that birds comple in fihite oscilation b/w pole and vehicle. but the total time and the total distance traveled by the the birds is fihite.

* Average velocity

$$\downarrow$$

$$\Rightarrow \frac{displacement}{total\ time\ taken}$$

$$\vec{V}_{AV} = \frac{\Delta \vec{s}}{\Delta \vec{t}}$$

* it is a vector quantity,
* and the direction of avg. velocity in time interval is same as disection of displacement in that interval.

magnitude avg. velocity $\Rightarrow |\vec{V}_{av}| = \frac{|\Delta \vec{s}|}{\Delta t}$

* $|\vec{V}_{av}| \leq$ avg. speed.

* in motion both conditions are true, it may happen
by both ways either observer moves or object moves.
this means it is in motion condition.

Ques:

the length of seconds hand is 20 cm. calculate the magnitude of avg. velocity of tip second hand for the time interval of 15s, 30s, 60's.

⇒

⇒ $2\sqrt{a^2}$

$0.2\sqrt{2}$

⇒ $\frac{0.2\sqrt{2}}{1S}$ ───── (i)

2) $\frac{0.4}{30}$ ───── (ii):
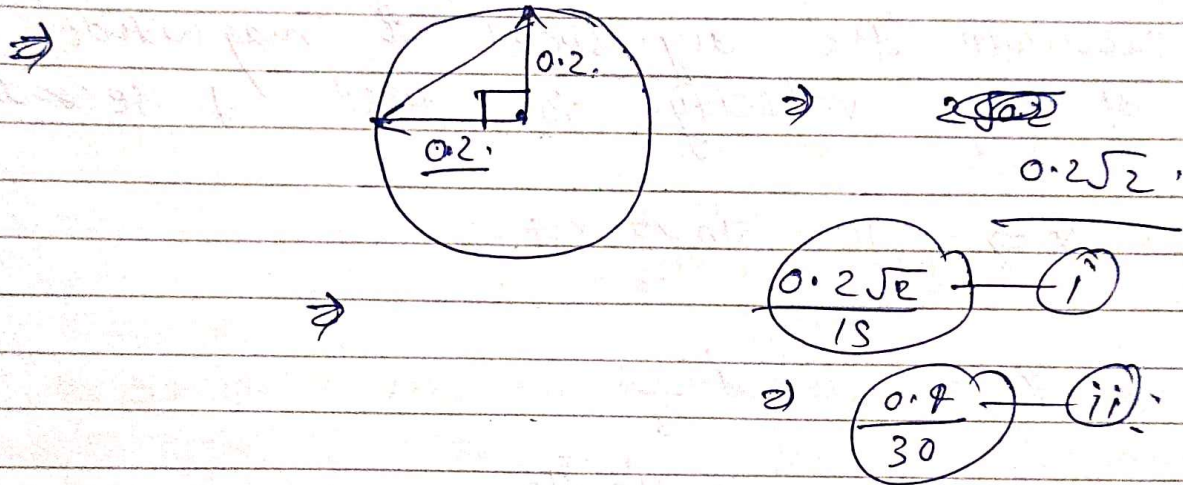
Ques:

X the x-co-ordinate of the particle moving along x-axis is given by.

$$x = t^2 + 2t$$

calculate the avg. speed & magnitude of avg. velocity for first 4 seconds.

⇒ $2t+2$ ⇒ (10 m)

⇒ ⇒ $16 + 8$ ⇒ $\frac{24}{4}$ ⇒ (6) m/s

⇒
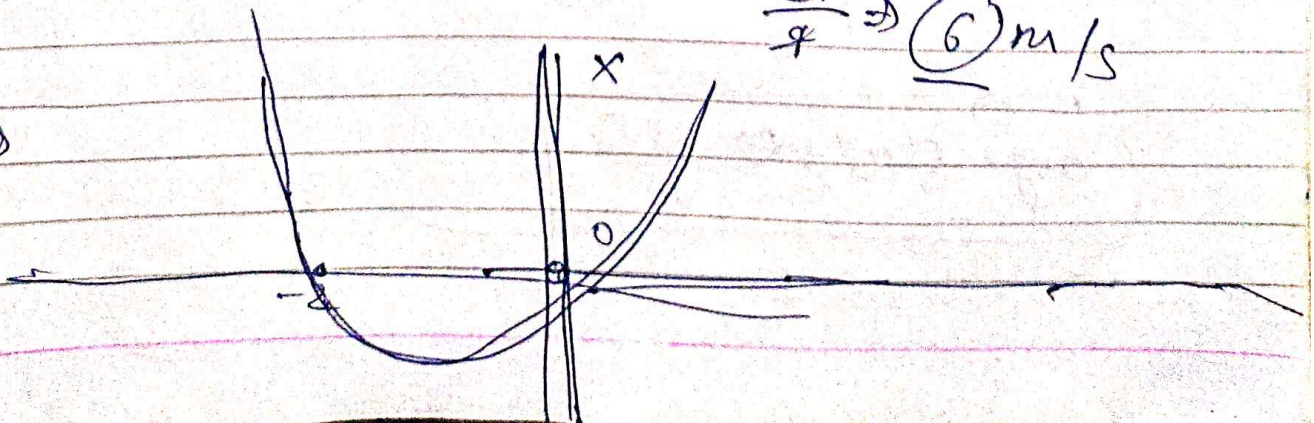
Ques.

X in straight line.

$$X = 10 \text{ cm} \sin\left(\frac{\pi}{6}t\right)$$

where t in s.

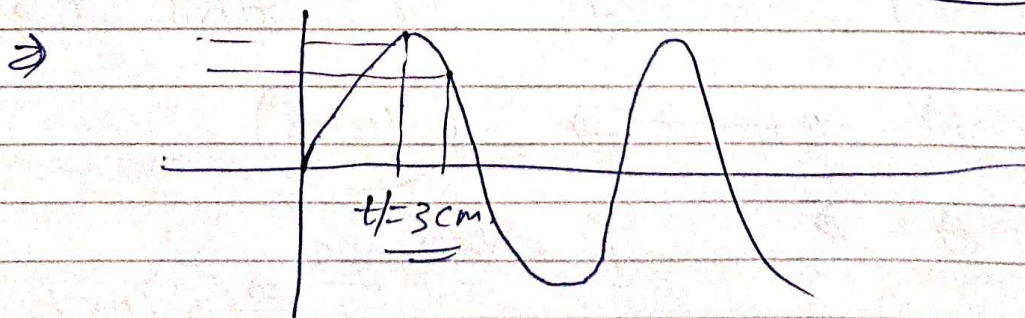calculate the avg. speed & magnitude of avg. velocity for first 9 seconds.

$$X \Rightarrow 10 \sin\left(\frac{\pi}{6} \times \frac{9}{3}\right)$$
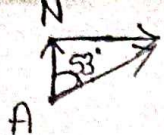
$$X = 10 \sin\frac{3\pi}{3}$$

$$v = \frac{10}{9} \sin \frac{2\pi}{3}$$

$$\Rightarrow 2.5 \sin \frac{2\pi}{3}$$

$$\Rightarrow \frac{10 \times \sqrt{3}}{9 \quad 2} \Rightarrow \boxed{\frac{5\sqrt{3}}{9} \text{ cm/sec.}}$$

$$\Rightarrow$$

t/=3cm

$$\boxed{\text{avg. speed} \Rightarrow \frac{20-5\sqrt{3}}{9}}$$

<u>Note</u>· N ↑53° → E   50 Km in the direction 53° E of N.
A

<u>Ques</u>:   A particle   travels   a   distance of 30 Km
towards north   and   followed by   a
distance   40 Km   east.   if the   total
time takes   the particle   in   its journey.
is   1 hour   then   calculate   the   avg· velocity
of the particle.

N

W   E

S
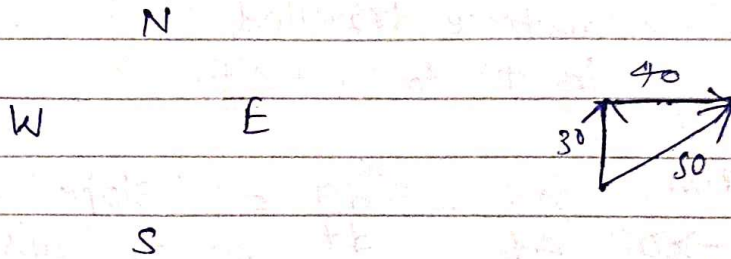
$$\tan Q = \frac{40}{30} = 53°.$$

50 Km in the direction 53° E of N°.

<u>Ques</u>. $X = 16t - 2t^2$.

$$\frac{dX}{dt} = 16 - 4t.$$

* the magnitude of instantaneous velocity and instantaneous speed are equal.
* the determination of Inst. velo. by using the definition usually involves calculation of derivative. we can find $V = \frac{dx}{dt}$ by using the standred results from differential calculus.

* ## Instantaneous speed and velocity.

differention of distance with respect to time is called Instantaneous speed.
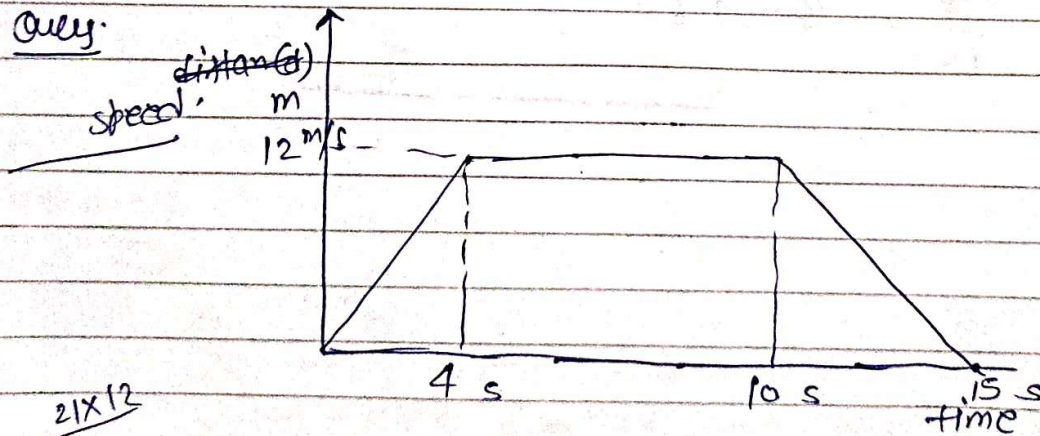
$t \rightarrow \Delta t$

$\Delta s \rightarrow$ distance travelled in t to $t + \Delta t$
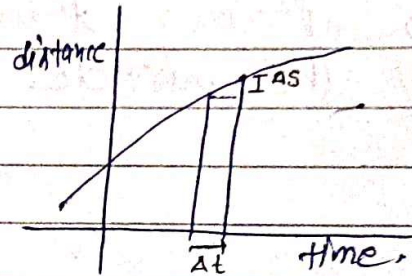
$\lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt} =$ slope of tangent drawn to distance vs time graph.

* distance travelled by a particle is equal to area occupied by speed v/s time graph of particle.

Quy:

$\dfrac{21 \times 12}{25 \, 2}$
$\dfrac{2}{2}$
$\textcircled{126}$

area
$\geq \boxed{126 \; m}$

$\Rightarrow \dfrac{1 \times 4 \times 12}{2} + 12 \times 6 + \dfrac{1}{2} \times 5 \times 12^{6}$

$24 + 72 + 30 \Rightarrow \boxed{126 \, m} \; \text{Ans.}$

only speed means Instantaneous speed.

for avg. speed we metioned in the question.

Ques.

$$S = t^2 + 2t$$

distance travelled ← | time (in s) →

calculate the instantaneous speed of the particle at $t = 2$ sec. also calculate the average speed for first two seconds.

$$\Rightarrow \text{Speed} = \frac{ds}{dt}.$$

$$= 2t + 2$$

at $t = 2$.

$$\text{Speed} = \boxed{6 \, m/s} \rightarrow \text{(instanteneous)} \atop \text{speed}$$

$$\Rightarrow \quad \text{avg. speed} \Rightarrow t^2 + 2t \Rightarrow (2)^2 + 2(2)$$

$$S \Rightarrow \quad 4 + 4 = 8 \, m$$

$$\text{avg. speed} = \frac{8}{2} \Rightarrow \boxed{4 \, m/s} \rightarrow \underline{avg.}$$

* use of instantaneous speed

distance travelled in $t = 2$ sec to

$t = 2.0001$ sec

$$\Rightarrow \qquad dv \times t = 6 \times 0.0001 = \underline{0.0006. \, m/s.}$$

⊙ another method.

$$\Rightarrow \quad S_{t=2} = \,'. \, 8 \, m$$

$$S_{t = 2.0001} = (2.0001)^2 + 2(2.0001)$$

$\Rightarrow$
$$dv = \frac{S_{t=2.0001} - S_{t=2}}{0.0001}$$

and same as $\Rightarrow$ 0.0006 m/s
for a particular instant. or small change in time.

Instantaneous velocity.

$$t \longrightarrow t + \Delta t$$

$$\Delta \vec{s}$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t} = \frac{d\vec{s}}{dt}$$

$$\boxed{\vec{v} = \frac{d\vec{s}}{dt} = \frac{d\vec{r}}{dt}} \longrightarrow \text{small change in position vector}$$

* rate of change of displacement / position
w.r.t time.

* for 1-D motion.

$$\vec{v} = \frac{dx}{dt} \hat{i}$$

$$v = \frac{dx}{dt}$$

direction of velocity is determined
by the sign of $\frac{dx}{dt}$.

* (tangent) Slope of x - t graph gives the instantaneous velocity

** Slope of tangent drawn to position - time graph is equal to instantaneous velocity.
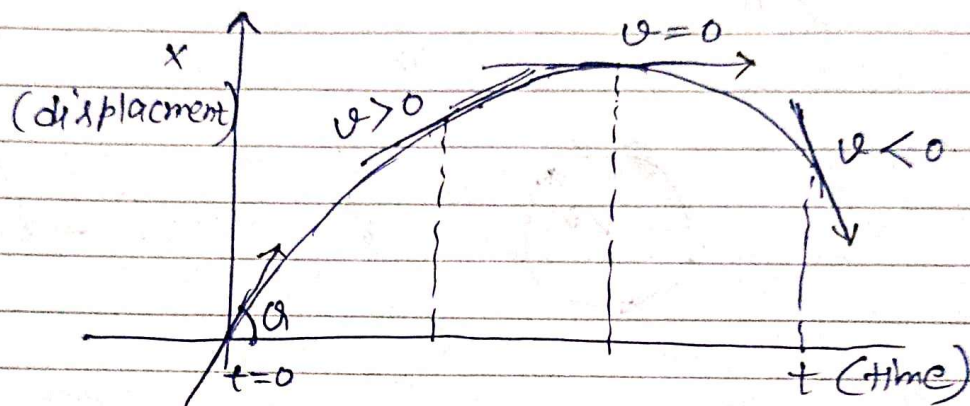
* Positive and negative instantaneous velocity. tell about only direction. (not actual -ve), in magnitude.

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{d\vec{s}}{dt}$$

$$|\vec{V}| = \left|\frac{d\vec{r}}{dt}\right| = \left|\frac{d\vec{s}}{dt}\right| \qquad \text{not written as } \frac{d|\vec{r}|}{dt}.$$

$$|\vec{V}| = \frac{|d\vec{r}|}{dt} = \frac{|d\vec{s}|}{dt} = \frac{ds}{dt} = \text{instantenous speed}$$

* for instantaneous time (very small time) magnitude of displacement is equal to distance and similarly velocity is equal to speed.
* Same in magnitude not in direction. because velocity has direction also.
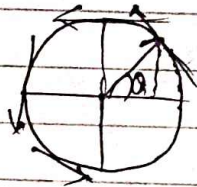
* The avg. velocity is equal to the slope of the line (chord) joining the points corresponding to P and Q on the x-t graph.

* direction of ↓ velocity of a particle is along tangent to the path of the particle, (means moves in +ve or -ve direction).

*
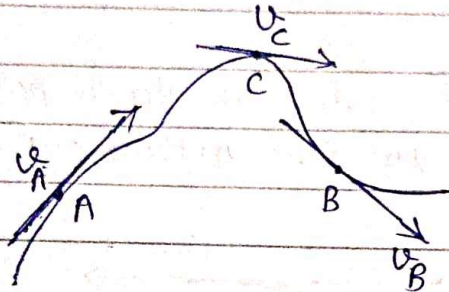
$$\vec{r} = r\cos\theta \,\hat{i} + r\sin\theta \,\hat{j}$$

$$\boxed{\dfrac{d(\vec{r})}{dt} = 0}$$

Ques.

$$\vec{r} = \left(\sqrt{t^2-1}\right)\hat{i} + \left(\sqrt{1+t^2}\right)\hat{j}$$

find velocity and speed of particle at time t.

~~right way~~

Sol.

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{2t}{2\sqrt{t^2-1}}\,\hat{i} + \frac{2t}{2\sqrt{t^2+1}}\,\hat{j}$$

$$|\vec{V}| = \sqrt{\frac{t^2}{(t^2-1)} + \frac{t^2}{(t^2+1)}}$$

$$= t\sqrt{\frac{2t^2}{(t^2-1)(t^2+1)}}$$

$$= \sqrt{\frac{2t^4}{t^4-1}}$$

$$|\vec{v}| = \frac{d|\vec{r}|}{dt}$$

$$|\vec{r}| = \sqrt{2}\, t$$

$$\frac{d|\vec{r}|}{dt} = \sqrt{2} \; m/s$$

Ques: a particle is moving along x-axis

$$x = t^2 - 4t \qquad\qquad x \to m$$
$$t \to s$$

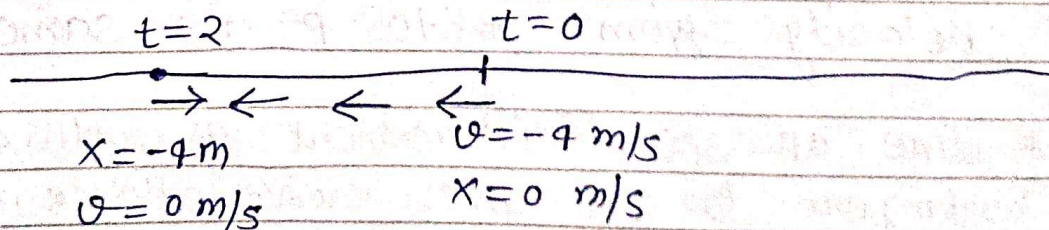find the time instant at which particle change in ~~distance~~ direction of motion. also find the position at that time.

Sol:

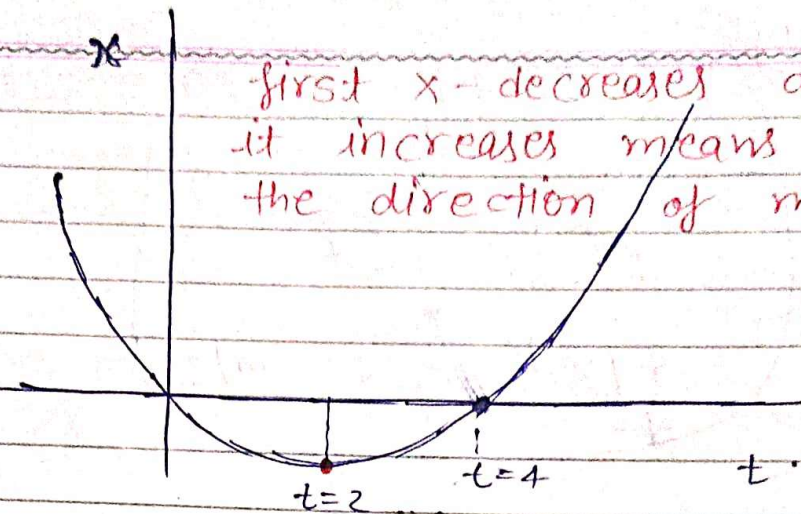$$v = \frac{dx}{dt} = 2t - 4 = 2(t-2)$$

$$0 < t < 2 \qquad\qquad v < 0$$

$$t > 2 \qquad\qquad v > 0$$

$$t = 2 \qquad\qquad t = 0$$

$$\rightarrow \leftarrow \leftarrow \leftarrow$$

$$x = -4m \qquad\qquad v = -4\,m/s$$
$$v = 0\,m/s \qquad\qquad x = 0\,m/s$$

first particle moves in -ve direction, and at $t = 2$ sec. particle change its dirction. and at that time particle cover 4m in -ve.

first x - decreases and at t=2
it increases means change
the direction of motion.

(tangent) slope at any points also gives the direction.

Ques. tell about any two point which has same direction of instantaneous velocity & aug. velocity in any path of particle,

A
O
slope.
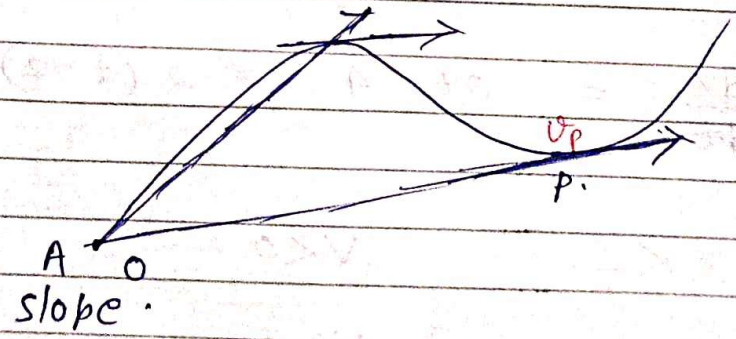
direction $\vec{v_p}$ and direction of aug. velocity from o to P are same.

* line oP shows displacement as well as tangent to the given curve. Hence, point P is the point at which direction of oP shows average as well as instantaneous velocity.

*

$$\int_{t_1}^{t_2} \vec{v} \, dt = \int_{\vec{r_1}}^{\vec{r_2}} d\vec{r}$$

$$\boxed{\int \vec{v} \, dt = (\vec{r_f} - \vec{r_i})}$$

integral of $\vec{v}$ w.r.t. time is equal to change in position - vector.

*   for   1-D motion.

$$x_f - x_i = \int v \, dt$$

*   area occupied by velocity time graph and time axis is equal to the displacement of particle.

*   if area lie above x-axis behave like +ve. but if area lie below the x-axis behave like -ve. and total subtract in each other. in velocity-time graph only. and gives the displacement.

*   But in case speed-time graph Area above, and below both treated as +ve. and add up each other. and give distance travelled by the particle.

in this question direction change in only 1 times at $t = 5\,s$.

$$\overrightarrow{\leftarrow}\overrightarrow{\leftarrow}\overrightarrow{\leftarrow}\overrightarrow{\leftarrow}\overrightarrow{\leftarrow}\overrightarrow{\leftarrow}\ldots v = 0$$

Ques *⊙

$\dfrac{10 \times 2}{3} = \dfrac{20}{3}\,m/s$

calculate the displacement & distance travelled by particle in 8 second.

* ⊙ in V-t graph if the particle above or below the axis, this means ~~does~~ ~~not~~ change in direction rather than any strictly up-down curve in any region.

⊙ but in X-t graph. if curve up and down it means particle changes its direction. rather than below or above or cut the axis. no mean change in direction.

eg.
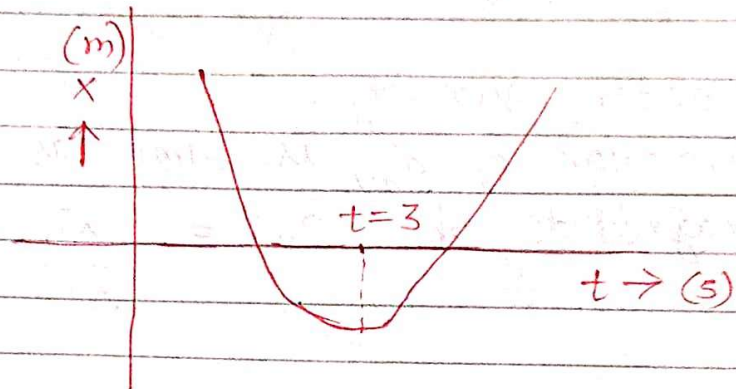
at t = 3 s it change the direction only one times.
because first curve decrease, and after t = 3 s curve increases.

eg.
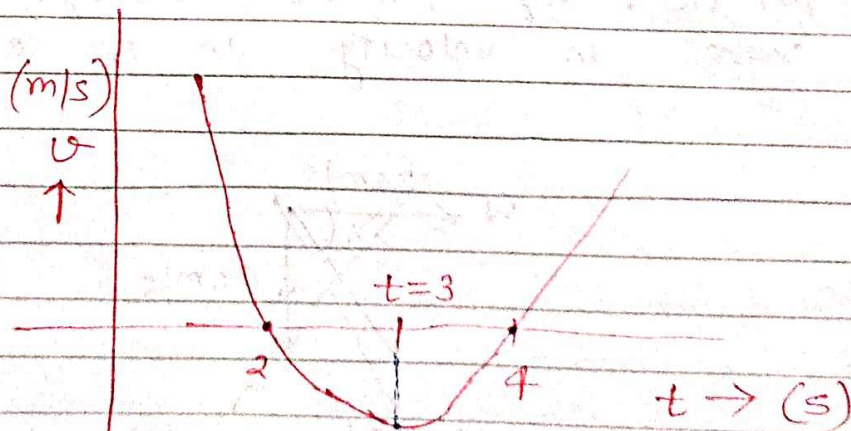
but in this case particle changes the direction 2 times. at t = 2 s and t = 4 s.
because before t = 2 s v is +ve. and after t = 2 s change the direction -ve. and thereafter once again change the direction at t = 4 s and +ve velocity of the particle.

* ## Acceleration

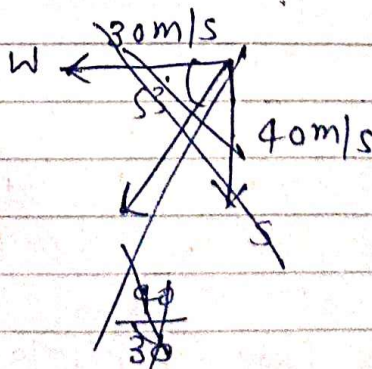$$\vec{a_{au}} = \frac{\Delta \vec{V}}{\Delta t}$$ Rate of change of velocity per unit time.

* Vector quantity.
* direction of $\vec{a}_{av}$ is same as direction of $\Delta \vec{V}$.
* magnitude of $\vec{a}_{av} = \dfrac{\Delta \vec{V}}{\Delta t}$

Ques: the particle moving with 30 m/s towards west takes a left turn and start moving towards south with speed 40 m/s. find the avg. acc. of the particle. if particle undergoes this change in velocity in 5 sec.
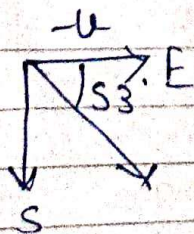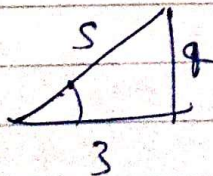
$\Rightarrow$ 53° south of ~~west~~ east

* avg. acc. is a vector quantity whose direction is same as that of the change in velocity.
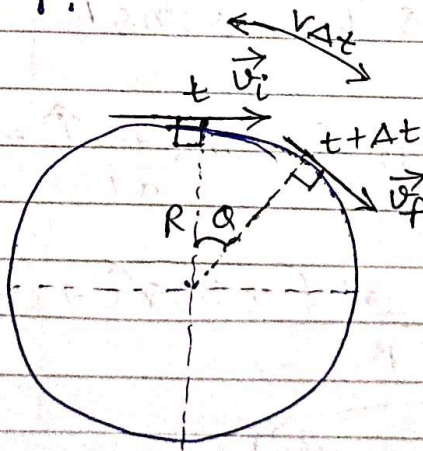
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* **Instantaneous acceleration**

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$
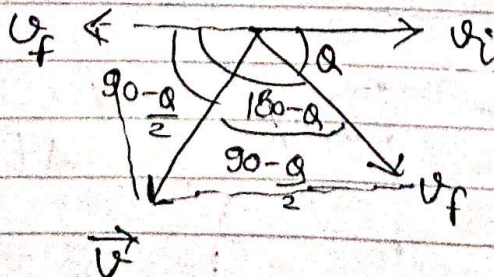
$$|\vec{a}| = \frac{|d\vec{v}|}{dt}$$

$$\Rightarrow \frac{d|\vec{v}|}{dt} \Rightarrow \text{rate of change of speed.}$$

Imp.

Ques. Calculate the magnitude & direction of Instantaneous acceler. of particle moving in circular path with constant speed v. of radius r.

$$\theta = \frac{v \Delta t}{R}$$

$$\vec{v}$$

$$|\overrightarrow{\Delta V}| = 2V\cos\left(90° - \frac{q}{2}\right) = 2V\sin\frac{q}{2}$$

$$|\vec{a}| = \lim_{\Delta t \to 0}\left|\frac{\Delta \vec{v}}{\Delta t}\right| = \lim_{\Delta t \to 0}\frac{|\overrightarrow{\Delta V}|}{\Delta t}$$

$$\Rightarrow \lim_{\Delta t \to 0} \frac{2V\sin\left(\frac{V\Delta t}{2R}\right)}{\frac{V\Delta t}{2R}} \cdot \frac{V}{2R}$$

$$= \lim_{} = \boxed{\frac{V^2}{R}}$$

$$\vec{V} = V\cos q \,\hat{i} - V\sin q \,\hat{j}$$

$$Q = \frac{Vt}{R}$$

$$\frac{dQ}{dt} = \frac{V}{R}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = V\left(-\sin q \frac{dq}{dt}\hat{i} - \cos q \frac{dq}{dt}\hat{j}\right)$$

$$= -\frac{V^2}{R}\left(+\sin q \,\hat{i} + \cos q \,\hat{j}\right)$$

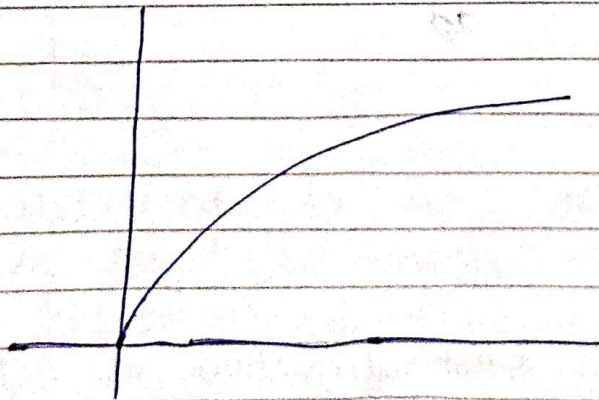$$\boxed{|\vec{a}| = \frac{V^2}{R}}$$

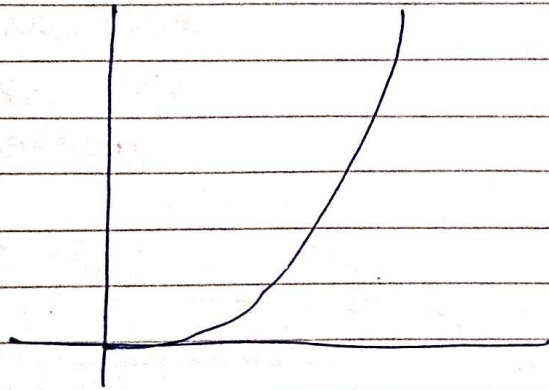for small time

$$\Delta t \to 0$$
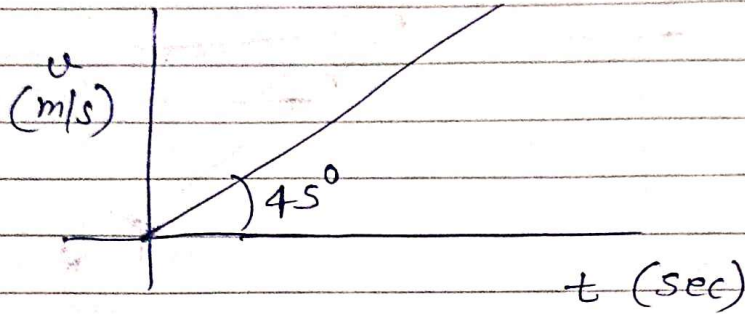
$$\frac{q}{2} = 0$$

90 - q/2
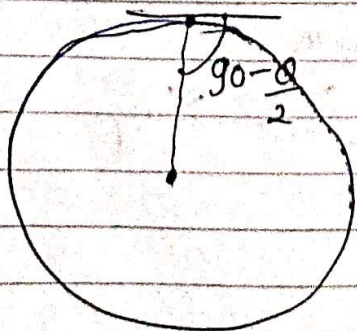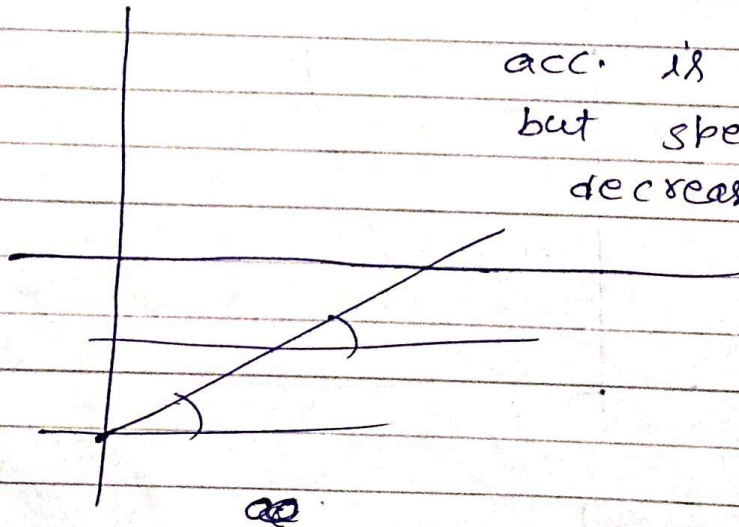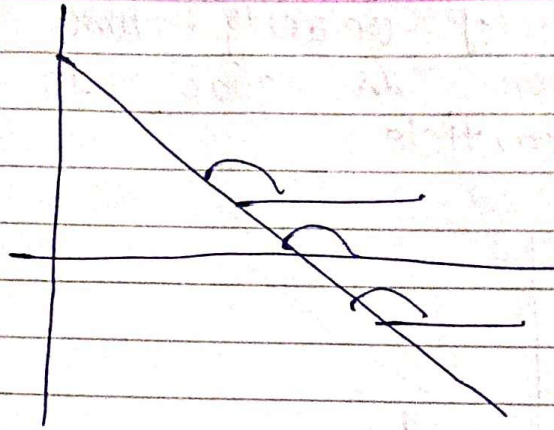
so, we says that

towards centre.

Note: at a free-fall condition, acc. depends on only on the forces which are applied on object that time.

* the slope of velocity - time graph for 1-D motion is equal to the acceleration of the particle

eg:

v
(m/s)

45°

t (sec)

-ve -ac.   +ve +ac

acc. is +ve
but speed is
decreassing.

∞

* if acc. is in same direction as velocity, then
  speed of the particle increases.

* if acc. is in oppositie direction to the velocity
  then speed decreases i.e. the particle slows
  down. this situation is known as retardation.

Retardation is nothing but it is slow down in speed, when the direction of acc. and velocity are opposite.

* $a = \dfrac{dV}{dt} = \dfrac{dV}{dS} \cdot \dfrac{dS}{dt}$

$$\boxed{a \Rightarrow \dfrac{V\, dv}{dS}}$$

* if we known acc. then we find velocity by integration.

$$a = \dfrac{dV}{dt}$$

$$\int_{t_1}^{t_2} a\, dt = \int_{v_1}^{v_2} dv$$

$$\boxed{V_2 - V_1 = \int_{t_1}^{t_2} a\, dt}$$

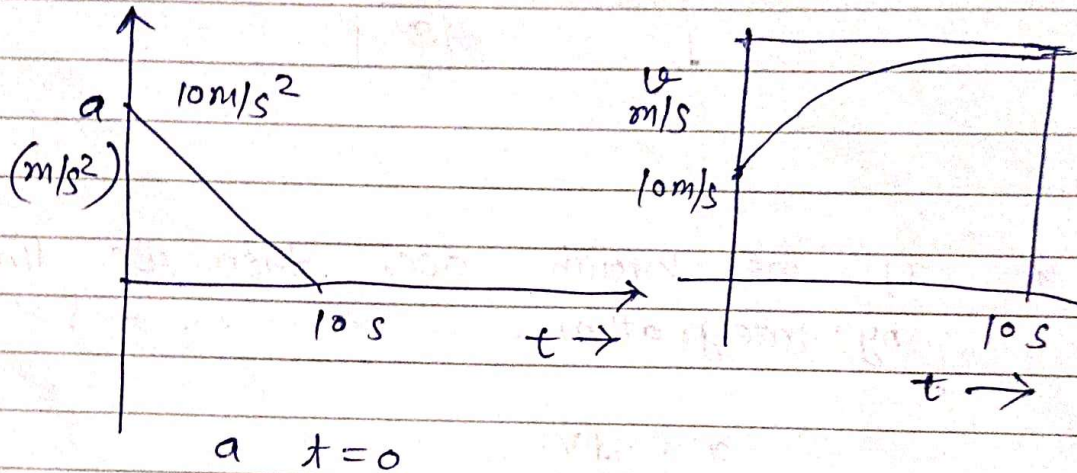* area occupied by $a-t$ graph and time axis is equal to change in velocity.

*

$$a = \dfrac{V\, dv}{dS}$$

$$\int_{S_i}^{S_t} a\, dS = \int_{V_i}^{V_f} V\, dv$$

$$\int_{S_i}^{S_f} a\, ds = \frac{1}{2} \left(v_f^2 - v_i^2 \right)$$

Ques.

X

$a \quad t=0$

the velocity of particle was 10 m/s.

find velocity at $t=10$ s and at general time 't'.

Solve.

$$v(t=10) - v(t=0) = \int_0^{10} a\, dt$$
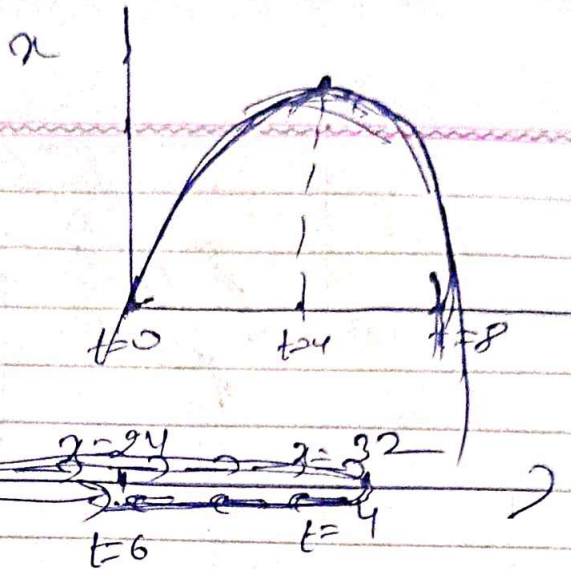
$$= \frac{1}{2} \times 10 \times 10$$

$$v - 10 = 50$$
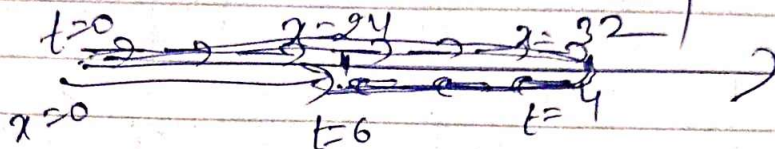
$$\boxed{v = 60 \text{ m/s}}$$

$$x = 16t - 2t^2$$

$$= 2t(8-t)$$

$$= 2 \times 4 (8-4)$$

$$= 8 \times 4$$

$$12(4)$$

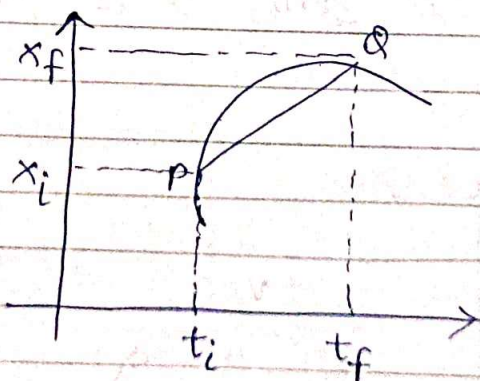$t=0$ $x=24$ $x=32$

$x=0$ $t=6$ $t=4$

✳ Graphical interpretation of some quantity.

① Avg. velocity.

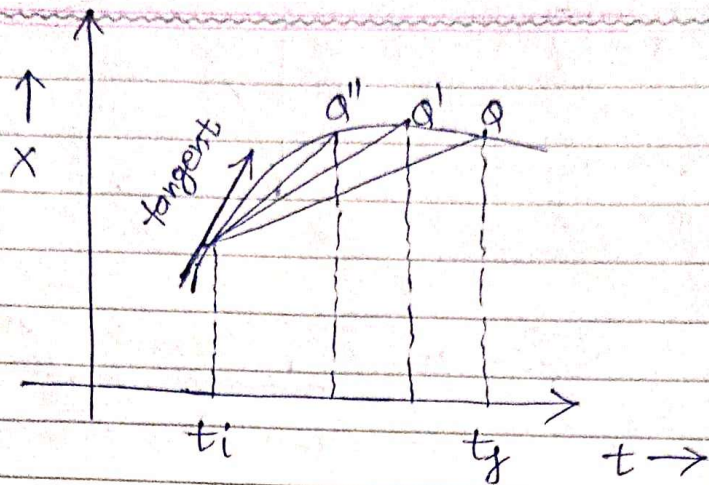   the avg. velocity is equal to the slope of the line (chord joining the points corresponding to P and Q on the x-t graph.

② instantaneous Velocity.

    as the point Q approaches P, the time interval approaches zero, but at the same time the slopes of the dotted line approaches that of the tangent to the curve at the point P.

as $\Delta t \rightarrow 0$, $V_{av} = \dfrac{\Delta x}{\Delta t} \rightarrow V_{inst.}$

as At → 0 , chord PQ → tangent at P.

Hence, the instantaneous velocity at P is the slope of the tangent at P in the x-t graph. when the slope of the x-t graph is +ve, v is +ve.
and slope is -ve, v is -ve,
and slope is zero, v is zero.
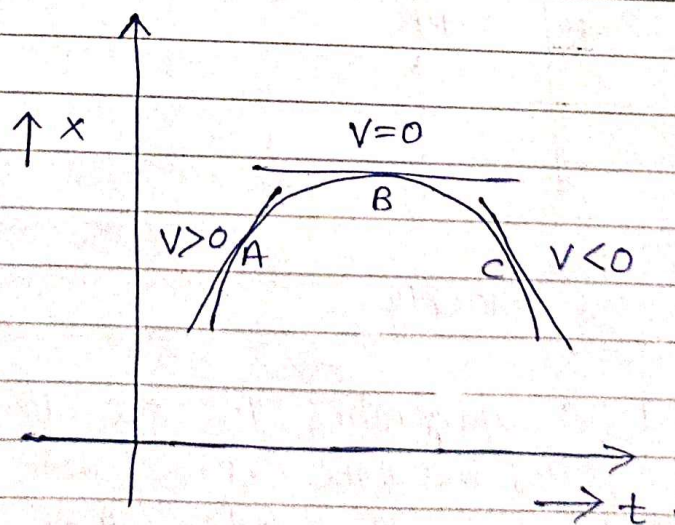
③　instantaneous　acceleration

the derivative of velocity with respect to time
is the slope of the tangent in velocity (v-t)
graph.

$\uparrow$ x

$a=0$

$a>0$

$a<0$

$a=0$

$a=0$

$t \rightarrow$
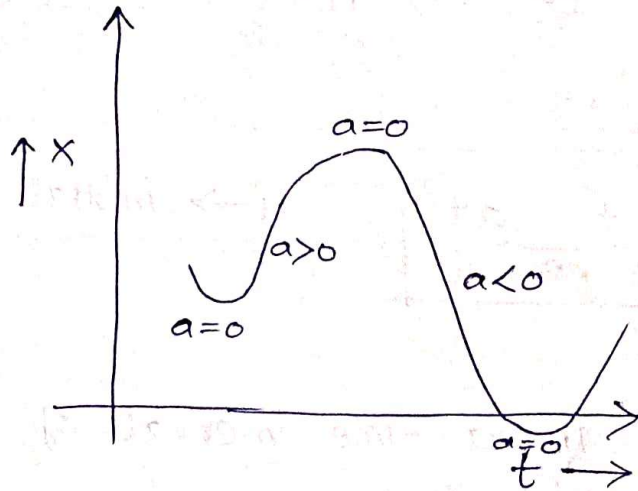
*　uniformly accelerated motion in straight line

$$v_f - v_i = a \int_{t_1}^{t_2} dt$$

$$v_f - v_i = at$$

$$\boxed{v - u = at}$$

v, u & a substitute
with proper sign.

$t \rightarrow$ time in sec

$v \rightarrow$ final velocity

$u \rightarrow$ initial velocity

* uniform motion ⇒ constant velocity
uniform acc. ⇒ constant acc.

* $S = \int v \, dt$

$$\Rightarrow \int_0^t (u + at) \, dt$$

$$\boxed{S \Rightarrow ut + \frac{1}{2} at^2}$$ $u \rightarrow$ initial velocity.

displacement in a time interval of t in a straight line.

* $a = v \dfrac{dv}{ds}$

$$\int_0^S a \, ds = \int_u^v v \, dv$$

$$as = \frac{v^2 - u^2}{2}$$

$$\boxed{v^2 - u^2 = 2as}$$

$S \rightarrow$ displacement in the interval in which velocity changes from u to v.

* ___Displacement in n^{th} seconds___

at n^{th} second. , $u \to$ initial velocity

$a \to$ acc.

$$S_n = un + \frac{1}{2} an^2$$

$$S_{n-1} = u(n-1) + \frac{1}{2} a(n-1)^2$$

$$S_n - S_{n-1} = \cancel{un} + \frac{1}{2} an^2 - \cancel{un} + u \, \underset{\cancel{2}}{\bullet} \, \cancel{an^2} \, \underset{2}{\bullet} \, \frac{a}{2} + \frac{2na}{2}$$

$$\boxed{S_n - S_{n-1} \Rightarrow u + \frac{a}{2}(2n-1)}$$

$\downarrow$

in u, t is multiplied which value $t=1$.

___Dimensionally___ correct.

___Ques.___ the particle is released from a height h above the ground surface. in the last Second of its motion it travels the distance of 25 m. calculate the height of tower.

___Ans.___

$$25 = 0 + 5(2n-1)$$

$$\boxed{n = 3 \, s}$$

$$S = 5 \times 3 \times 3$$

$$\boxed{S \Rightarrow 45 \, m}$$

Ans.

Ques.

$a = a$
$u = 2u$

A •— ·— — — — — —

B •— — — — — — — —
$u = u$
$a = 2a$

find the distance from initial position at which 'B' overtakes 'A'.

Ans.

let the time t.

$\Rightarrow$ $2ut + \dfrac{1}{2}at^2 = ut + \dfrac{1}{2}(2a)t^2$

$ut = \dfrac{1}{2}at^2$

$t = 0 , \quad t = \dfrac{2u}{a}$

Ques. a car is moving on a highway with a speed of 72 Km/h. the breaks of a car produce max. deacceleration 4 m/s². at what min. distance. from an obstacle. so that the car comes to rest.

what would be the problem in ans. if the car was traveling at twice the initial speed.

Ans.

$$(0)^2 = (20)^2 + 2(-4)s$$

$$s = \frac{400}{8} \Rightarrow \boxed{50\,m}$$

$$s = \frac{20\times40}{2\times4} \Rightarrow \boxed{200\,m}$$

if the max. retardation due to slipary is 2 m/s²

Ques. a train can move with max. acc. of 8 m/s². and it can retard with max. rate of 4 m/s². this train straits from rest ● from station A, and rest at station B. which s km away from A. if the train cover this distance in minimum time. find the max. ve speed which the train can acquired.

Ans.

$\Rightarrow$

after time t.

$\Rightarrow$

$v = 8t$ $\Rightarrow$ $S = 0 + \dfrac{8t^2}{2}$

$\Rightarrow$ $\cancel{2}(8t)^2 = \cancel{2} 2 \times 4 \times S$

$64 t^2 = 8S$

$$\boxed{S = 8t^2}$$

$\Rightarrow$ $4t^2 + 8t^2 = 5000$

$12 t^2 = 5000$

$t^2 = \dfrac{2500}{6}$ ✓

$t^2 = \dfrac{1250}{3}$

$t = \sqrt{\dfrac{1250}{3}}$

Que. a ballon is moving upward width a const. velocity of 10 m/s. when the ballon at a height of 15 m above the ground. a stone is realeased from the ballon. find the time after which the stone strike surface.
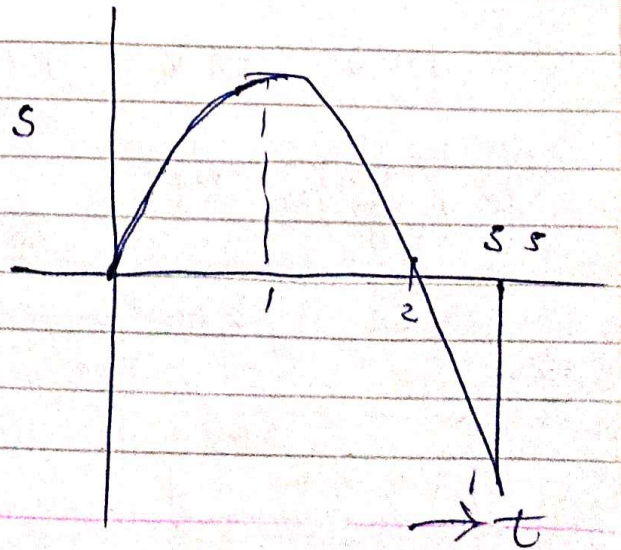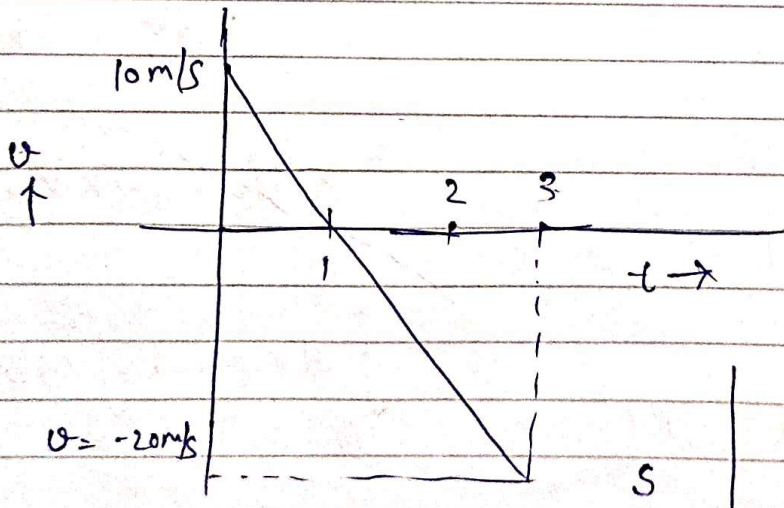
$$Q = -10t + 5t^2 - 15$$

$$\Rightarrow \frac{10 \pm \sqrt{100 + 300}}{10}$$

$$\Rightarrow \frac{10 \pm 20}{10} \Rightarrow \boxed{3\ s}$$

$$\underline{v^2 - u^2 = 2as}$$

$$\Rightarrow v^2 = 100 + 20 \times 15$$

$$\Rightarrow 100 + 300 \qquad \Rightarrow \boxed{20\ m/s}$$

10 m/s

$v \uparrow$

2   3

1

$t \rightarrow$

$v = -20 m/s$

S

5 s

1

2

1

$\rightarrow t$

* Ques. a particle is travelling with velocity V_0. at $t=0$. a force starts acting on particle which produce a retardation of KV. K is +ve const. and $V \to$ inst. velocity find the total distance travelled by the particle during retardation. also find the velocity at time t.

Sol.

$$a = -KV$$

$$\frac{dV}{dt} = -KV \qquad\qquad \frac{V\,dV}{ds} = -KV$$

$$\int_{V_0}^{V} \frac{dV}{V} = -k\int_{0}^{t} dt \qquad\qquad \int V\,dV = -KV\,ds$$

$$\int_{V_0}^{0} dV = -K\int_{0}^{S} ds$$

$$\ln V - \ln V_0 = -kt$$

$$\ln\left(\frac{V}{V_0}\right) = -kt \qquad\qquad \Rightarrow (0 - V_0) = -KS$$

$$\Rightarrow \frac{V}{V_0} = e^{-kt} \qquad\qquad \boxed{S = \frac{V_0}{K}}$$

$$\boxed{V = V_0 e^{-Kt}}$$

$$S = \int v \, dt$$

$$S = \int_{0}^{\infty} v_0 \, e^{-kt} \, dt$$
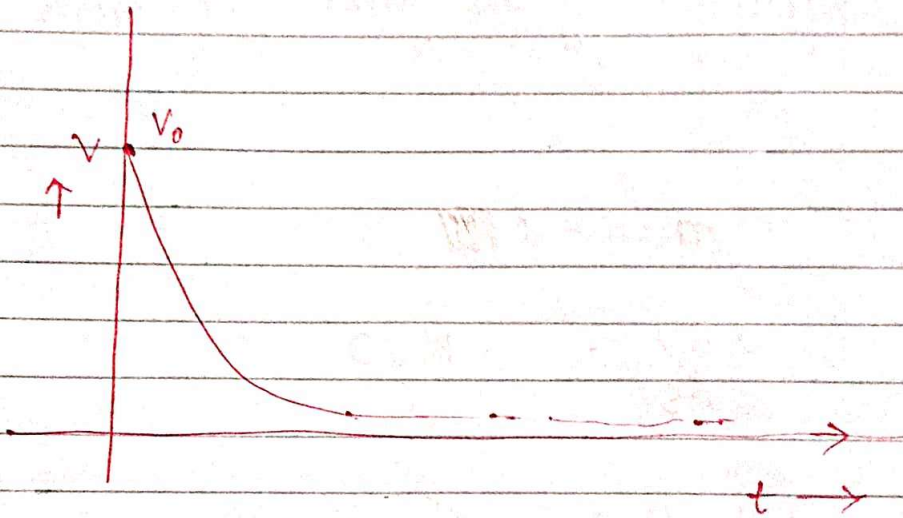
$$\Rightarrow \left[\frac{v_0 \, e^{-kt}}{-k} \right]_{0}^{\infty}$$

$$\Rightarrow \frac{-v_0}{k} (0 - 1)$$

$$\boxed{S = \frac{v_0}{k}}$$

Ques. a particle is moving with velocity V_0. a force acts on the particle which produces a retardation. which is directly proportional to $\sqrt{S}$. where S is the displacement of the particle. calculate the avg. velocity of the particle by the time it comes to rest.

Sol

$$a = -K\sqrt{S}$$

$$V\frac{dV}{dS} = -K\sqrt{S}$$

$$\int_{V_0}^{0} V\,dV = -K\int_{0}^{S} \sqrt{S}\,dS$$

$$\frac{1}{2}\left(0^2 - V_0^2\right) = -K\frac{S^{3/2}}{3/2}$$

$$\frac{V_0^2}{2} = \frac{KS^{3/2}}{3/2}$$

$$S^{3/2} = \frac{3}{4}\frac{V_0^2}{K}$$

$$S = \left(\frac{3V_0^2}{4K}\right)^{2/3}$$

$$S = \left(\frac{3}{4K}\right)^{2/3} V_0^{4/3}$$

$$\frac{1}{2}\left(v^2 - v_0^2\right) = \frac{-2}{3} K S^{3/2}$$