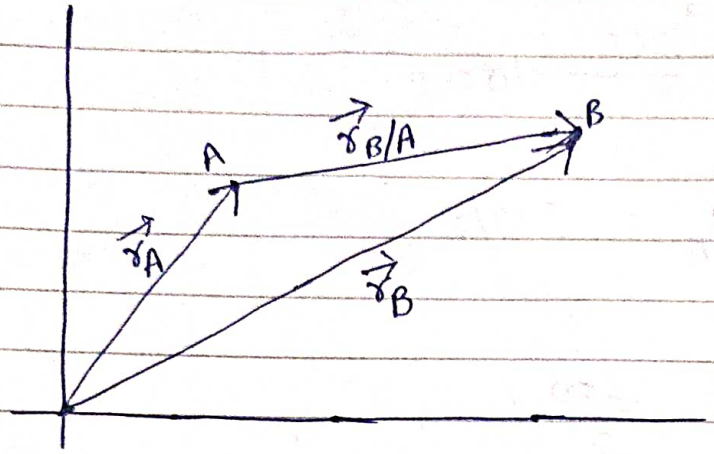
Relative motion



$$\vec{r}_A + \vec{r}_{B/A} = \vec{r}_B$$

$$\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$$

$$\boxed{\vec{r}_{A/B} = -\vec{r}_{B/A} = \vec{r}_A - \vec{r}_B}$$

$$\vec{v}_{AO} = \frac{d\vec{r}_{AO}}{dt} \qquad \vec{v}_{A/B} = \frac{d}{dt}\vec{r}_{A/B}$$

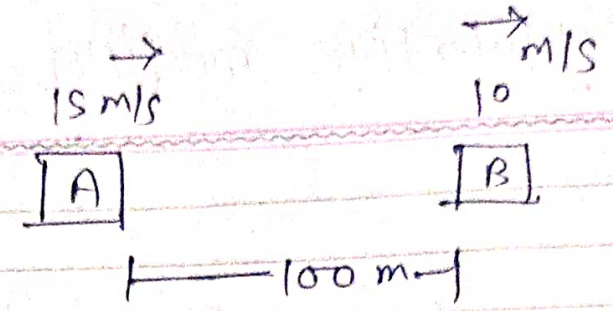
$$= \frac{d}{dt}\left(\vec{r}_A - \vec{r}_B\right)$$

$$\Rightarrow \frac{d\vec{r}_A}{dt} - \frac{d\vec{r}_B}{dt}$$

$$\boxed{\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B}$$

$$\boxed{\vec{a}_{A/B} = \vec{a}_A - \vec{a}_B}$$

Ques. 1.

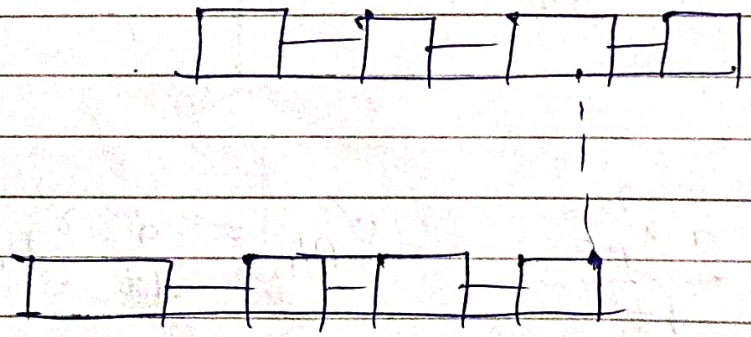


$$v_A - v_B = 5 \text{ m/s}$$

$$t = \frac{100}{5}$$

$$t = 20 \text{ s} \quad \text{Ans.}$$

Ques. 2.



$v_A = 54 \text{ km/h}$
$\Rightarrow 15 \text{ m/s}$

$v_B = 90 \text{ km/h}$
$\Rightarrow 25 \text{ m/s}$

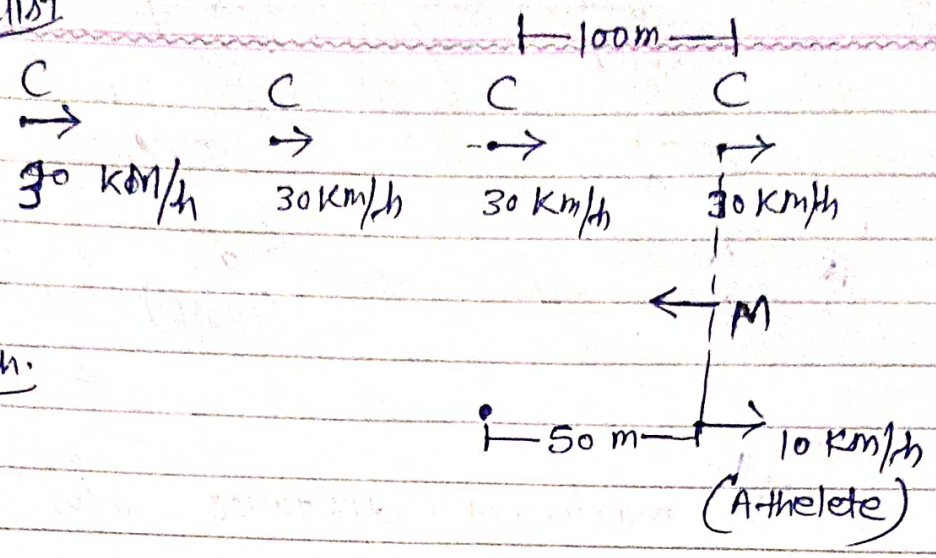
$$l_A = l_B = l$$

Train B passes in time 10 s.

$$\Rightarrow v_{B/A} = v_B - v_A = +10 \text{ m/s}.$$

$$\Rightarrow l = v_{B/A} \times t. \Rightarrow 100 \text{ m}.$$

Ques. 3.



what velocity of the men in opposite direction. when he meet the men. also meet the cyclist.

50 + 30 × t

$\vec{v}_c - \vec{v}_m$

$\Rightarrow \quad 30 - (-v)$

$\Rightarrow \quad 30 + v$

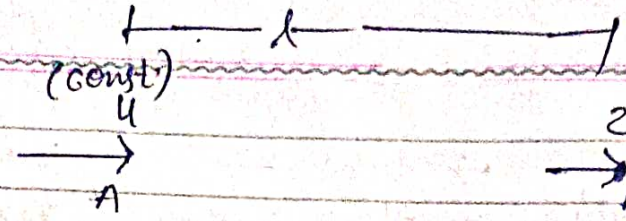
$$\frac{100}{30+v} = \frac{50}{10+v}$$

$$20 + 2v = 30 + v$$

$$v = 10 \text{ Km/h}$$

Ans.

Ques.



$\vec{a}_B \rightarrow$ const.

= a towards left

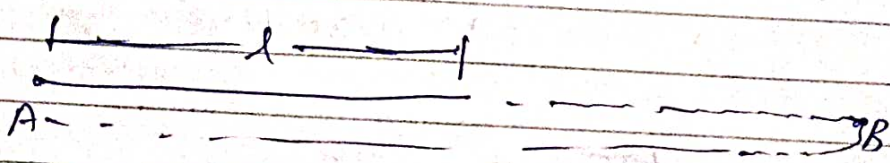
find the maximum distance b/w A & B

find the time after meet A.

Sol: ⇒ $v_{B/A}$ ⇒ $2u - u = u$ m/s

⇒ $\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

⇒ $= -a - 0 = -a$



when $2u \longrightarrow$ to

distance decreases.

$$v_{re}^2 = u_{re}^2 + 2a_r s_{re}$$

$$0 = u^2 + 2(-a)s_r$$

$$\boxed{s_{rel} = \frac{u^2}{2a}}$$

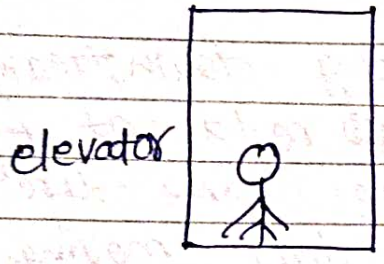
$$d_{max} = l + \frac{u^2}{2g}$$

$$s_{rel} = u_{rel}\, t + \frac{1}{2} a_{rel}\, t^2$$

$$s_{rel} = -l.$$

$$-l = ut - \frac{1}{2} at^2$$

Ques.

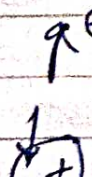


$\downarrow a = 1 m/s^2$

a coin is dropped inside the lift from certain height above the floor of the lift. the coin strikes the lift in 1 sec. Calculate the height from which the coin was dropped.

Soln,

$$h = s_{rel} = u_{rel}\, t + \frac{1}{2} a_{rel}\, t^2 \uparrow$$



$$\Rightarrow 0 + \frac{1}{2}(10-1)(1)^2$$

$$h = 4.5\ m$$

Ques Solve the above problem if the lift is accelerating vertically upward direction.

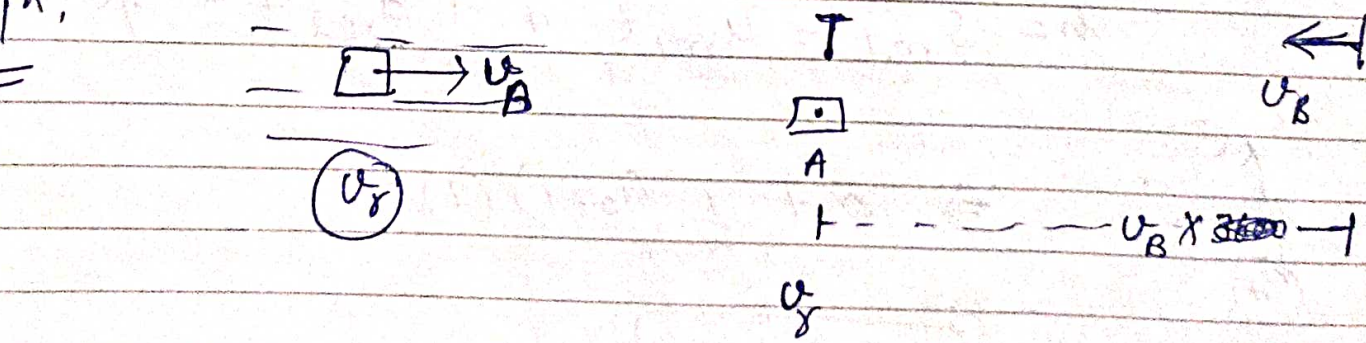
$$\Rightarrow h = S_{rel} = u_{rel}\,t + \frac{1}{2} a_{rel}\,t^2$$

$$\Rightarrow 0 + \frac{1}{2}(10+1)(1)^2.$$

$$\Rightarrow \boxed{5.5\ m}\ \text{Ans.}$$

Ques. There is a motorboat going downstream overcomes (passes) through a ~~~~ raft at pt. A the boat continue to move same direction for 60 min. and then it start moving in upstream direction. maintaining the same speed with respect to river. it again crosses the raft at a point which is 6 km ~~from~~ away from the pt. A. find the velocity of river current.

Soln.



$$\Rightarrow 2t = 60$$

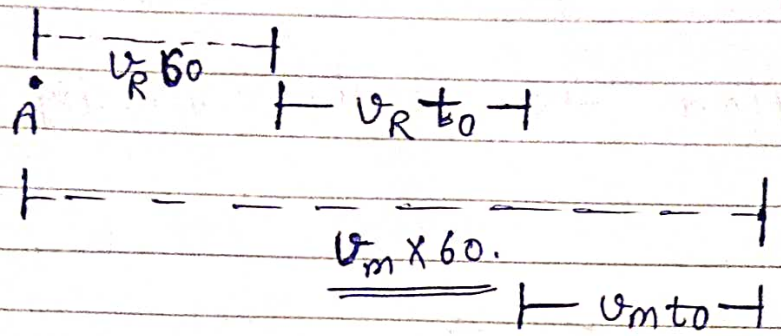
$$\Rightarrow \boxed{3\ \text{KM/h}}$$

Ans

(VR)

$v_m = V_B - V_R$

$V_B = V_m + V_R$



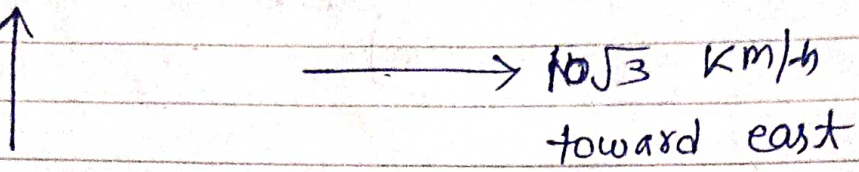
* Relative ~~velocity~~ motion in two Dimensions

$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$$

$$\vec{a}_{A/B} = \vec{a}_A - \vec{a}_B$$

North

Ques. 10 Km/h



find the relative velocity.

Sol^n.

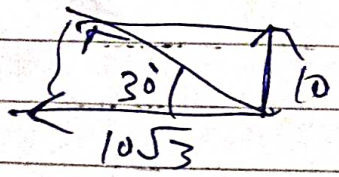
$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$$

$$= 10\,\hat{j} - 10\sqrt{3}\,\hat{i}$$

$$\Rightarrow -10\sqrt{3}\,\hat{i} + 10\,\hat{j}$$

$$\Rightarrow \sqrt{(10\sqrt{3})^2 + (10)^2}$$

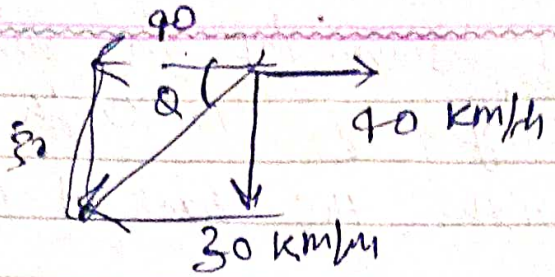
$$\left|\vec{v}_{A/B}\right| \Rightarrow 20 \text{ Km/h}$$



$$\tan\theta = \frac{10}{10\sqrt{3}} \Rightarrow \frac{1}{\sqrt{3}}$$

$\theta = 30^\circ$ North of west

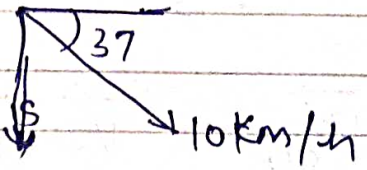
Ques. ✗ There is a flag on the car. Now a car is moving with a velocity of 40 Km/h towards east. & and wind is blowing on this reagion with a velocity of 30 Km/h towards south. Find the direction in which the flag will wave.



⇒ ⇒ 50.

⇒ $\tan = \frac{30}{40} \Rightarrow 37^\circ$ S of W.

Ques a motar boat is moving in a river with a velocity of 10 km/h. in the direction 37° S of east. with respect to the river.



if the velocity of river current is 5 km/h towards south. find the speed and the in which the boat appear to move to stationary on the ground.

Soln.

$$v_B = \vec{V}_{B/R} - \vec{V}_R \Rightarrow 10 \text{ km/h}$$

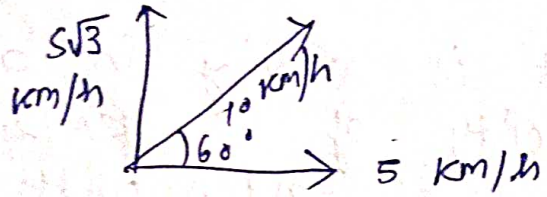
$$V_R = 5 \text{ km/h}$$

$$V_B = 8\hat{i} - 11\hat{j}$$

$$|\vec{V}_B| = \sqrt{121+64} = \sqrt{185}$$

$$\theta = \tan^{-1}\frac{11}{8}$$

Ques. a swimmer is swimming in still rivers with a velocity of 10 km/h in the direction 60° with the bank of river (N of E). with what velocity should a man move along the bank of the rivers so that the ~~man~~ swimmer to move in the dirn ⊥ to bank of the river



$\rightarrow v_m$

$\rightarrow i$

$$\vec{v}_{s/m} = \vec{V_s} - \vec{V_m}$$

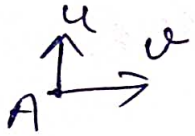
$$\Rightarrow \left(5\hat{i} + 5\sqrt{3}\hat{j}\right) - V_m\hat{i}$$

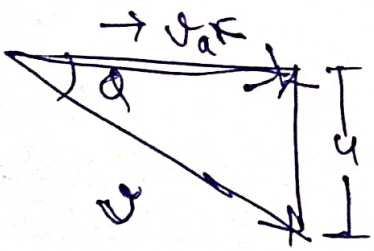
$$\Rightarrow (5 - V_m)\hat{i} + 5\sqrt{3}\hat{j}$$

$\Rightarrow$

Deer an aeroplane need to fly city A to B. the air speed of plane is v. now there exist a const. wind in the direction ⊥ to the line AB. and the velocity of wind is u. find the dirⁿ in which the plane must maintain air speed so that it can ~~fly~~ fly A to B. also find the time of flight of A to B. distance is l.

Sol.ⁿ.





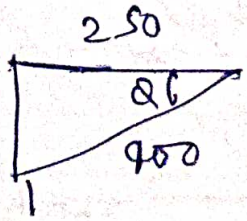
$$\theta = \sin^{-1}\left(\frac{u}{v}\right).$$

400 ←

Ques.

an aeroplane maintain an air speed of 400 km/h towards west after flying for 30 min. it find itself at a pt. which is 250 km towards west & 30 km towards south from the starting pt. find the velocity of wind ~~from~~ in that region.

400 ←



$$\tan\theta = \frac{30}{250}$$

10/07/16

$$\vec{S_a} = 250\hat{i} + 30\hat{j}$$

$$\Rightarrow \vec{v_a} \times \frac{1}{2} = 250\hat{i} + 30\hat{j}$$

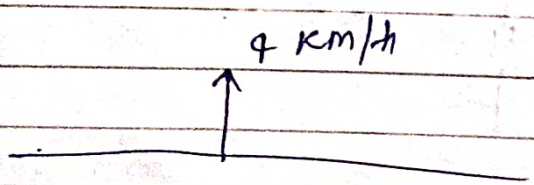
$$\vec{v_a} = 500\hat{i} + 60\hat{j}$$

$$400\hat{i} + V_w =$$

$$V_w = 100\hat{i} + 60\hat{j}$$

* Man - river Problems.

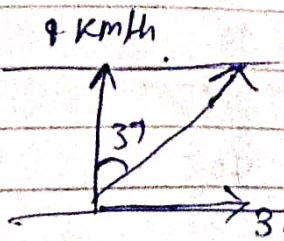
Ques.



a man swims with a velocity of 4 km/h in the dirn ⊥ to the river flow.

If the velocity of river is 3 km/h. and the width of the river is 500 m. find the drift of the man by the time he crosses the river.

Soln.



$$\cos 37 = \frac{B}{h}$$

$$\Rightarrow \Rightarrow \frac{500 \times 3}{4 \times 3}$$

$$\Rightarrow \frac{1}{2 \times 4} = \frac{1}{8}\ h$$

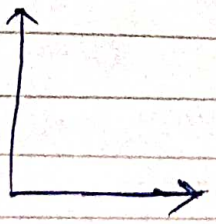
drift $\Rightarrow 3 \times \frac{1}{8} \Rightarrow \frac{3}{8}$ km

* $\vec{v}_{MR} + \vec{v}_R = \vec{v}_M$

$\vec{v}_{MR}$: velocity of man in still water (or) velocity of man w.r.t. river (or) velocity with which man swims.

$\vec{v}_M$: velocity of man w.r.t. ground.

Qus. in a river the velocity of river current is 5 km/h. man starts one bank of the river and crosses the river Laruly. if $V_{MR} = 10$ km/h. find the dirn in which must be swimming.



$10 \cos\theta = 5$

$\cos\theta = \frac{1}{2}$

$\theta = 60°$

Ques. In the above problem. Calculate the time in which man cross the river. if the width of the river is.

$$\text{time} = \frac{1 \text{ km}}{.5\sqrt{3} \text{ km/h}}$$

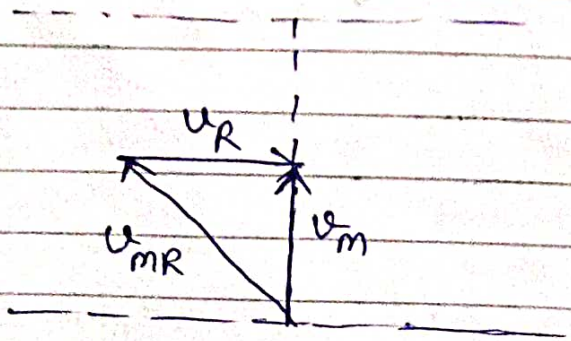
$$\Rightarrow \frac{1}{5\sqrt{3}} \text{ hr} \rightarrow \text{Ans.}$$

Imp derivation

Ques. a man can swim in still water $v_{mR} = 5$ km/h and the $v_R = 3$ km/h. find the time it will take for the man to cross a river of 500 m width. if width ~~the~~ of the river w. then.

i) he crosses the river in shortest path.

ii) he crosses the rive min. time.



$$d_{min} \Rightarrow \sqrt{v_{mR}^2 - v_R^2}$$

$$\text{time} = \frac{w}{\sqrt{v_{mR}^2 - v_R^2}} \Rightarrow \frac{1/2 \text{ km}}{\sqrt{5^2 - 3^2}}$$

$$\Rightarrow \frac{1}{8} \text{ hrs.}$$

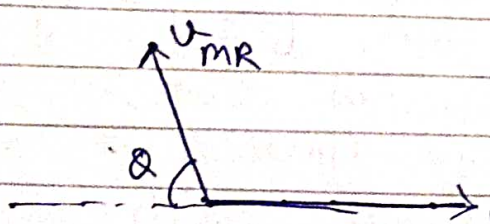
$$\text{Time} = \frac{dy}{v_y}$$

$$t_{min} \Rightarrow \frac{w}{v_{mR}\sin\theta}$$

$\frac{1}{10}$ hr $\Rightarrow$ min. time

Ques. $v_{mR} = 4$ km/h in still water and $v_R = 5$ km/h. find the angle with which angle man must swim. so that drift is minimum.

Sol^n.



drift

$$\text{drift} = 0$$

$$v_R - v_{mR}\cos\theta = 0$$

$$\cos\theta = \frac{v_R}{v_{mR}} = \frac{5}{R}$$

~~dir~~ drift can not be zero.

$$\text{drift} = (v_R - v_{mR}\cos\theta) \times \frac{w}{v_{mR}\sin\theta}$$

$$\frac{d}{d\theta}(\text{drift}) = 0$$

$$\frac{\omega}{v_{MR}}\left(\sin\theta(+v_{MR}\sin\theta) - (v_R - v_{MR}\cos\theta)\cos\theta\right) = 0$$

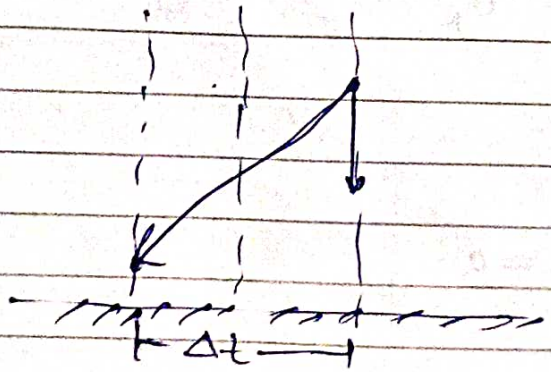
$$v_{MR}\sin^2\theta - v_R\cos\theta + v_{MR}\cos^2\theta = 0$$

$$v_{MR} = v_R\cos\theta$$

$$\cos\theta = \frac{v_{MR}}{v_R} \Rightarrow \theta = \cos^{-1}\left(\frac{v_{MR}}{v_R}\right) =$$

$$\Rightarrow \cos^{-1}\left(\frac{4}{5}\right) = 37^\circ$$

* Rain - man Problems.

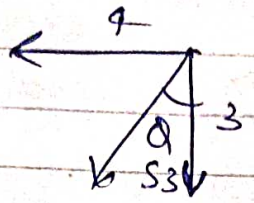


Ques. Raindrops are falling vertically with a velocity of 3 km/h.
a man is walking on horizontal surface with velocity 4 km/h.
find the velocity of rain w.r.t man.

Soln.

$$\vec{v}_{Rm} = \vec{v}_R - \vec{v}_m$$

$$= -3\hat{j} - 4\hat{i}$$

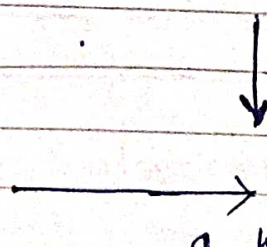


5 km/hr

53° with the vertical.

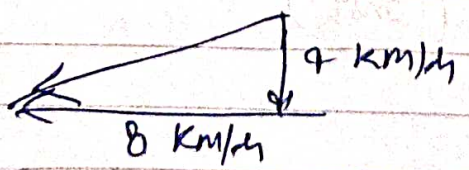
$$\cos^{-1}\left(\frac{4}{5}\right) = 37^\circ$$

Ques. To a men moving with velocity of 4 km/h on a horizonal surface. rain drops appears to fall vertically. with a velocity of 4 km/h. if men increases his velocity in the same direction to 8 km/h find the direction in which rain drops appear to fall him.



⇒

⇒



$$\sqrt{16+64} \Rightarrow \sqrt{80}$$

$$\Rightarrow 4\sqrt{5} \text{ m/s}.$$

⇒

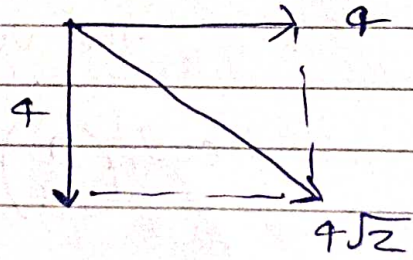
$$\tan\theta = \frac{4}{8} \frac{1}{2}$$

$$\theta = \tan^{-1}\frac{1}{2} \rightarrow \text{Ans.}$$

Sol^{n}: $\vec{v}_{Rm} = \vec{v}_R - \vec{v}_m$

$-4\hat{j} = \vec{v}_R - (4\hat{i})$

$\vec{v}_R = 4\hat{i} - 4\hat{j}$



$\Rightarrow \quad \vec{v}_m = 8\hat{i}$

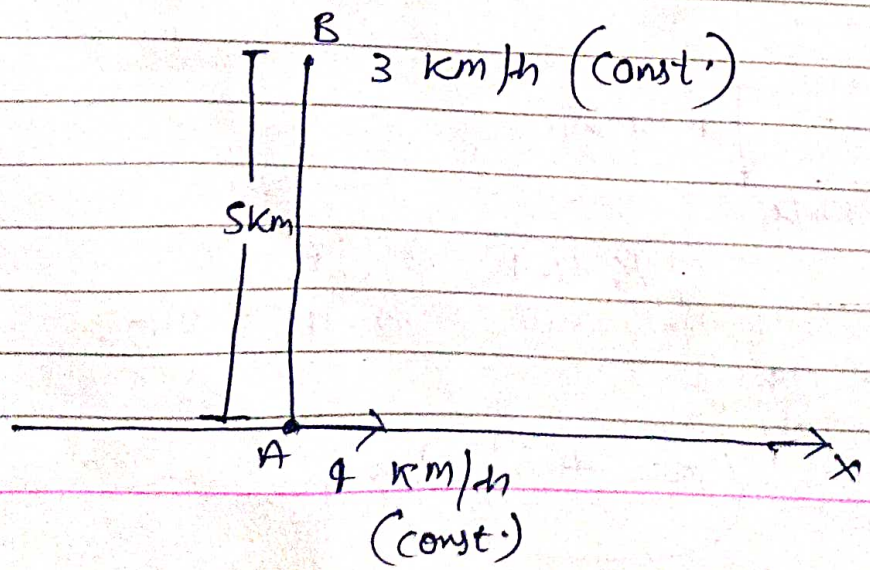
$\vec{V}_{Rm} = \vec{v}_R - \vec{v}_m$

$\Rightarrow 4\hat{i} - 4\hat{j} - 8\hat{i}$

$\Rightarrow -4\hat{i} - 4\hat{j}$.

~~Qu~~ Collision, sepration & approach

eg.



find the minimum sepration b/w A & B also find the time . at which sepration b/w them will be minimum.

Soln.

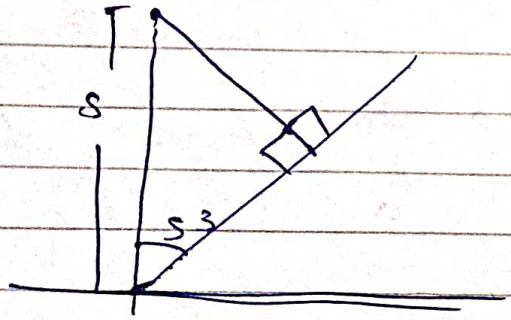
$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$$

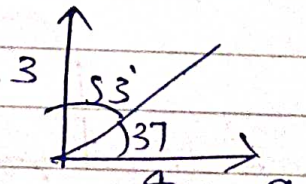
⇒ path of A w.r.t will straigh line.

$$\vec{v}_{A/B} = 4\hat{i} - (-3\hat{j}) = 4\hat{i} + 3\hat{j}$$

$$d_{min.} = 5 \sin 53^\circ$$

$$= 4 \text{ km}$$





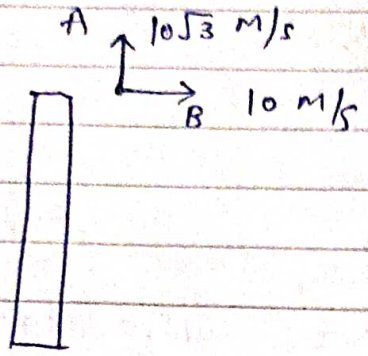
$$s_{A/B} = 3 \text{ km}.$$

$$t = \frac{s_{A/B}}{v_{A/B}} = \frac{3}{5} \frac{\text{km}}{\text{km/h}}$$

$$\Rightarrow \frac{3}{5} \text{ hr.}$$

Ans.

Ques.



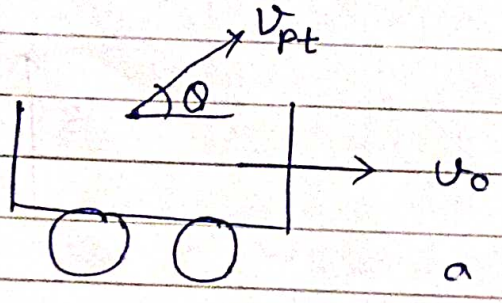
A & B are thrown simultaneously

Q⇒ find the separation b/w A & B after 2 s.

⇒

$= 20 \times 2 + \Rightarrow 40$ m Ans.

Ques.



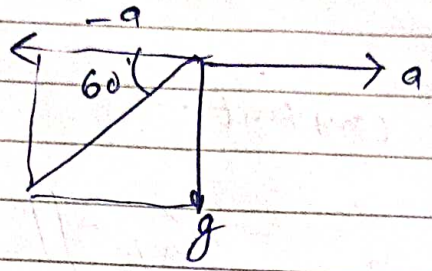
⇒ $v_{pt} = v_p$

what must be acc. of the truck so that the path of the particle w.r.t. truck is st. line.

and hence, find the time after which the particle returns back to to the truck.

$$\vec{a}_{ball/truck} = \vec{a}_{ball} - \vec{a}_{truck}$$

$\Rightarrow$



$$\tan 60 = \frac{g}{a}$$

$$a = \frac{g}{\sqrt{3}}$$

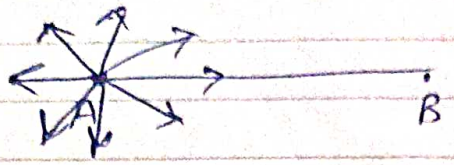
$\Rightarrow$ $\dfrac{2 v_{rel.}}{a_{rel}}$

$\Rightarrow$ $\dfrac{2 \times 100}{\sqrt{g^2 + \frac{g^2}{3}}}$

$\Rightarrow$ $\dfrac{2 \times 100}{\frac{2 \times 10}{\sqrt{3}}}$ 10

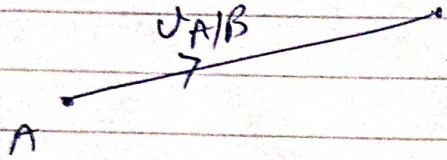
$T \Rightarrow 10\sqrt{3}$ sec Ans.

Ques.



$\vec{v}_{A/B} \Rightarrow$ constant.

$\vec{v}_{A/B} \parallel \vec{r}_{B/A}$.



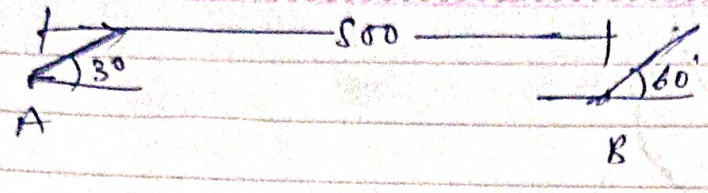
(or)

$$\frac{\vec{v}_{A/B}}{|\vec{v}_{A/B}|} = \frac{\vec{r}_{B/A}}{|\vec{r}_{B/A}|}$$

$$\frac{\vec{v}_A - \vec{v}_B}{|\vec{v}_A - \vec{v}_B|} = \frac{\vec{r}_B - \vec{r}_A}{|\vec{r}_{B/A}|}$$

(component of $\vec{v}_{A/B}$ in the direction $\perp^{or}$ to line joining A & B must be zero.

Ques. two plane A & B are situated 500 m, velocity of A make an angle 30° with the horizontal. and velocity of B makes angle 60° with the horizontal. the velocity of A with respect to B is $\perp$ to the direction of motion of A & the velocity of A is $100\sqrt{3}$ m/s. find the time after which they collide.



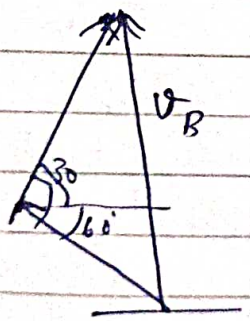
$\vec{v}_{A/B} \perp \vec{v}_A$

$\vec{v}_{A/B} + \vec{v}_B = \vec{v}_A$

$\vec{v}_{A/B} \perp \vec{v}_A$

$\vec{v}_{B/A} \perp \vec{v}_A$

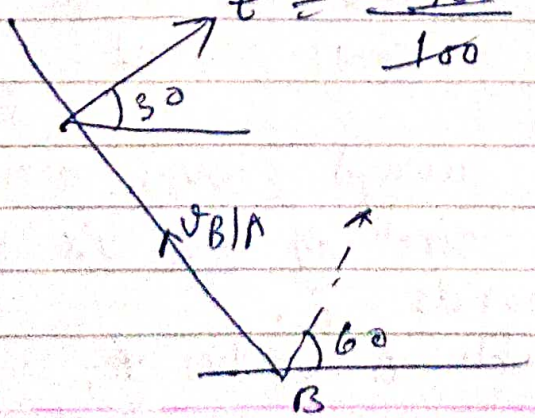
$\vec{v}_{B/A} + \vec{v}_A = \vec{v}_B$



$\tan 60^\circ = \frac{v_A}{v_{B/A}}$

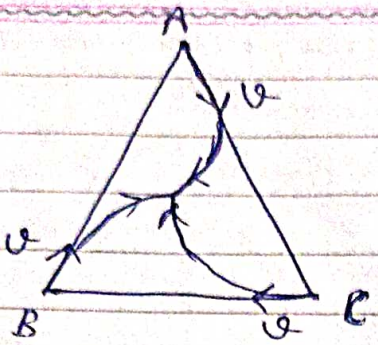
$v_{B/A} \Rightarrow \frac{100\sqrt{3}}{\sqrt{3}}$

$\vec{v}_{B/A} = 100$ m/s.

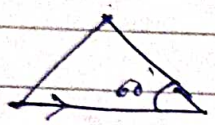


$t = \frac{500}{100} \Rightarrow 5$ sec.

Ques.



$$v_{sep} = \frac{l}{v_{Approach}}$$



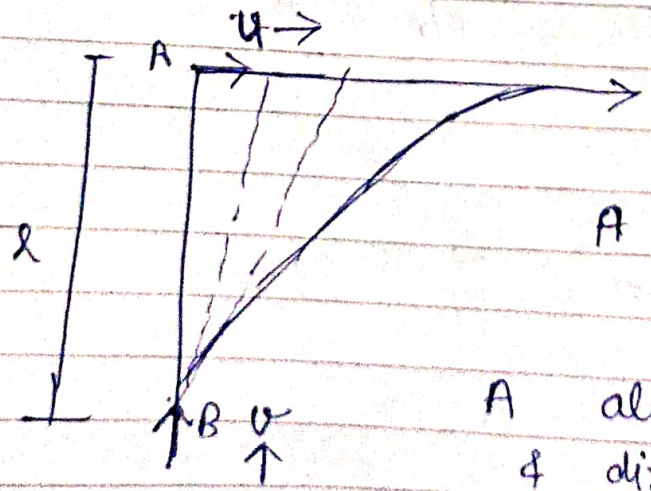
$$v_{app} = v + v\cos 60°$$

$$v_{app} = \frac{3v}{2}$$

$$\Rightarrow \frac{3v}{2} t = l$$

$$\boxed{t = \frac{2l}{3v}}$$

Ques.



A and B move with Const. speed.

A always moves along x-axis & direⁿ of v_B is always towards A.

find the time which B catches A.

Soln. ⇒

$$V_{app} = v - u\cos\theta$$

$$\int_0^T (v - u\cos\theta)\, dt = l$$

$$\int_0^T v\, dt - \int_0^T u\cos\theta\, dt = l$$

$$vt - u\int_0^T \cos\theta\, dt = l$$

$$uT = \int_0^T V\cos\theta\, dt$$

$$uT = V\int_0^T \cos\theta\, dt.$$

$$\int_0^T \cos\theta\, dt = \frac{uT}{V}$$

$$VT - \frac{u.uT}{V} = l$$

$$T = \frac{V.l}{v^2 - u^2}$$

Ques in the above problem find the initial acc. of B.

$$\vec{v}_B = v\cos\theta\,\hat{i} + v\sin\theta\,\hat{j}$$

$$\vec{a}_B = \frac{d\vec{v}_B}{dt} = v\left(-\sin\theta\frac{d\theta}{dt}\,\hat{i} + \cos\theta\frac{d\theta}{dt}\,\hat{j}\right)$$

$$\Rightarrow \vec{a}_B)_{t=0} = -v\times l\times\frac{d\theta}{dt}\,\hat{i}$$

$$\boxed{\frac{d\theta}{dt} = \frac{u}{l}}$$

$$\Rightarrow -v\times l\times -\frac{v}{l}\,\hat{i}$$

$$\Rightarrow \frac{vu}{l}\,\hat{i}$$