Projectile motion

(Two dimensional motion)

(or)

motion in a plane

$$\boxed{\vec{r} = x\hat{i} + y\hat{j}}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \left(\frac{dx}{dt}\right)\hat{i} + \left(\frac{dy}{dt}\right)\hat{j}$$

$$\boxed{\vec{v} = v_x\hat{i} + v_y\hat{j}}$$

$$v_x = \frac{dx}{dt}$$

$$\int dx = \int v_x\,dt$$

$$x_f - x_i = s_x = \int v_x\,dt$$

$$v_y = \frac{dy}{dt}$$

$$y_f - y_i = s_y = \int v_y\,dt$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(v_x\hat{i} + v_y\hat{j}\right)$$

$$\vec{a} = \left(\frac{dv_x}{dt}\right)\hat{i} + \left(\frac{dv_y}{dt}\right)\hat{j}$$

$$\boxed{\vec{a} = a_x\hat{i} + a_y\hat{j}}$$

$\int \vec{v}\, dt$ = displc. $= \int d\vec{s}$

$\int |\vec{v}|\, dt$ = distance $= \int |d\vec{s}|$

$a_x = \frac{dv_x}{dt}$

$\int a_x\, dt = \Delta V_x$

$a_y = \frac{dv_y}{dt}$

$\int a_y\, dt = \Delta V_y$.

** **Note:**

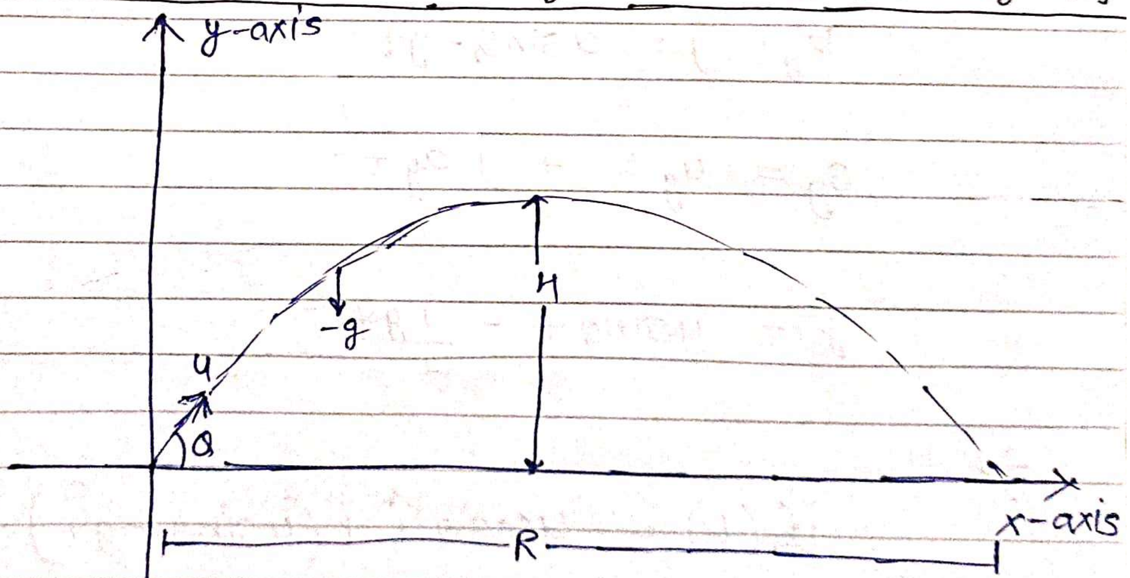
$\vec{a} \Rightarrow$ constant.

and $\vec{u} \times \vec{a} \neq 0$

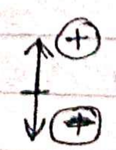
then the path will be parabola

* A stone projected an angle θ with the horizontal

*



$a = g$.



$\Rightarrow \vec{a} = -g\,\hat{j}$

$a_x = 0$

$v_x = u\cos\theta$

$v_y = u\sin\theta$

$\vec{v} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

*

$v_x(t) = v_x$

$v_x(t) = u\cos\theta$.

$s_x = v_x \times \text{time}$

$x - 0 = u\cos\theta \times t$

$\boxed{x(t) = u\cos\theta\, t}$ $\Rightarrow$ Range if $t \to T$

$\Rightarrow$

$a_y = -g$

$v_y = u\sin\theta$

$v_y(t) = u\sin\theta - gt$

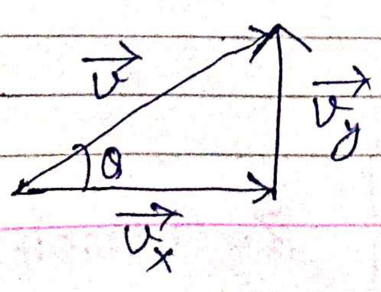
$s_y = u_y t + \frac{1}{2} a_y t^2$

$y = u\sin\theta\, t - \frac{1}{2} g t^2$

$\Rightarrow$

$\vec{v}(t) = u\cos\theta\, \hat{i} + (u\sin\theta - gt)\,\hat{j}$

$|\vec{v}(t)| = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$



* direction of particle at any time (we find ϕ at t)

*
$$\tan\phi = \frac{u\sin\theta - gt}{u\cos\theta}$$

* Height from horizontal surface.

$$v_y^2 = u_y^2 + 2a_y S_y$$

$$(0)^2 = (u\sin\theta)^2 + 2(-g)H$$

$$H = \frac{u^2\sin^2\theta}{2g}$$

$$H = \frac{u_y^2}{2g}$$

at $\theta = 90^\circ$ H will be maximum

$$H_{max} = \frac{u^2}{2g}$$

* Time of flight (T)

In 'T' time - $S_y = 0$

$$0 = u\sin\theta\, T - \frac{1}{2}gT^2$$

$T = 0$, $T = \dfrac{2u\sin\theta}{g}$

at $\theta = 90^\circ$ T will be maximum

$$T_{max} = \frac{2u}{g}$$

* **Range.**

$R = S_x$ in 'T' time

$= u\cos\theta T$

$= (u\cos\theta)\dfrac{2u\sin\theta}{g}$

$$R = \frac{2 v_x v_y}{g}$$

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

at $\theta = 45^\circ$

Range will be max. for a given velocity.

$$\boxed{R = \frac{u^2}{g}}$$

Ques: the particle is projected from the ground at an angle θ. max. height attain by the particle is 20 m. find the time of particle

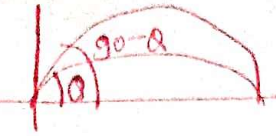
Ans.

$$20 = \frac{u^2 \sin^2\theta}{2g}$$

$u\sin\theta = 20$

$$T = \frac{2u\sin\theta}{g} \Rightarrow \frac{2 \times 20}{10} \Rightarrow 4\,s \text{ Ans}$$

*



R is same in both cases.

Ques. a particle is projected from ground with speed u at angle θ with the horizontal. such that the kinetic energy of the particle at max. height is 1/2 of its initial kinetic energy. then find the angle of the projection.

Ans. 60°. ⇓ Ans.

$\frac{1}{4} \times \frac{1}{2} m u^2 = \frac{1}{2} \times m \times u^2 \cos^2\theta$

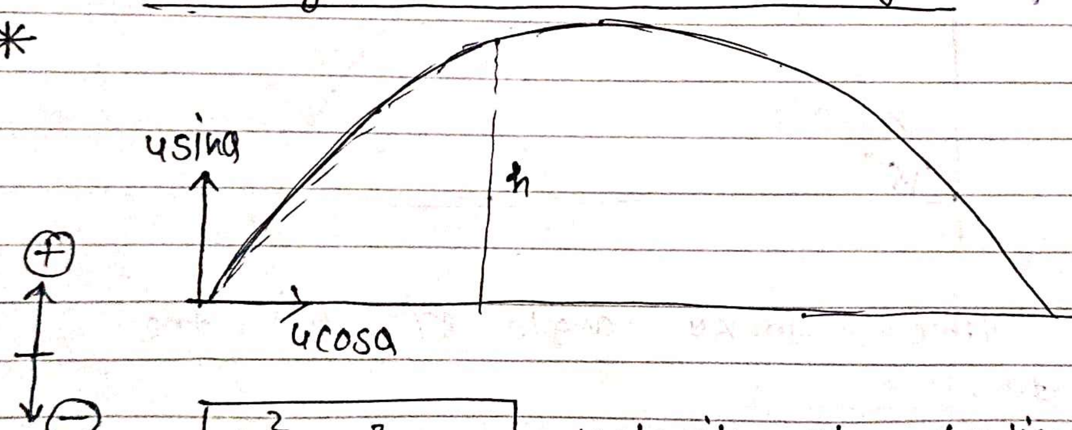
at a max. height $v_y = 0$ so only v_x component present.

$\cos\theta = \frac{1}{2}$

$(\theta = 60°)$

* velocity at a particular height.

*



$\boxed{v^2 = u^2 - 2gh}$ velocity at a particular height

$v_y^2 = u^2\sin^2\theta + 2(-g)(h)$

$\Rightarrow v^2 = v_y^2 + u^2\cos^2\theta \qquad (v_x = u\cos\theta)$

$v^2 \Rightarrow u^2\sin^2\theta - 2gh + u^2\cos^2\theta$

$\boxed{v^2 = u^2 - 2gh}$

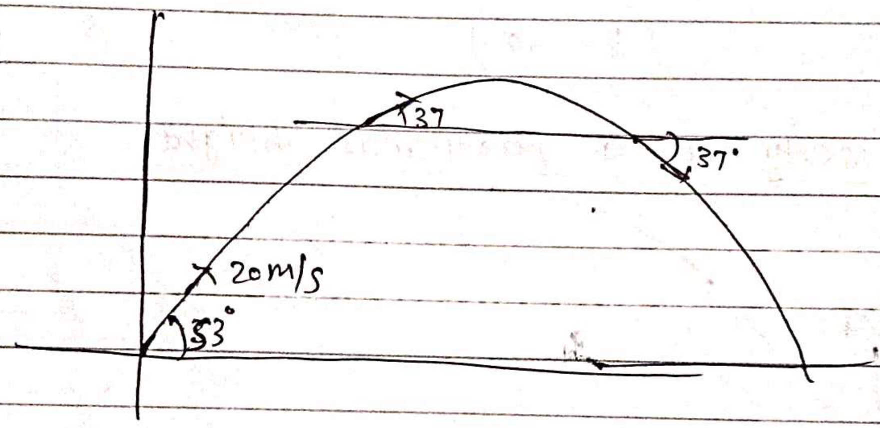
* basic ~~reason~~ reason of this principle is conservation of energy.

$$\frac{1}{2} m u^2 = mgh + \frac{1}{2} m v^2$$

$$u^2 = 2gh + v^2$$

$$\boxed{v^2 = u^2 - 2gh}$$

Ques.



which time make angle 37 with the horizontal.

Ans. $t = 0.9\ s$ → Ans.
$t = 2.5\ s$

$$\tan\phi = \frac{u\sin\theta - gt}{u\cos\theta}$$

$$\Rightarrow \tan 37 = \frac{20\sin 60^\circ - 10t}{20\cos 60^\circ}$$

$$\Rightarrow \frac{10\sqrt{3} - 10t}{10} = \frac{3}{4}$$

$$\Rightarrow \sqrt{3} - t = \frac{3}{4}$$

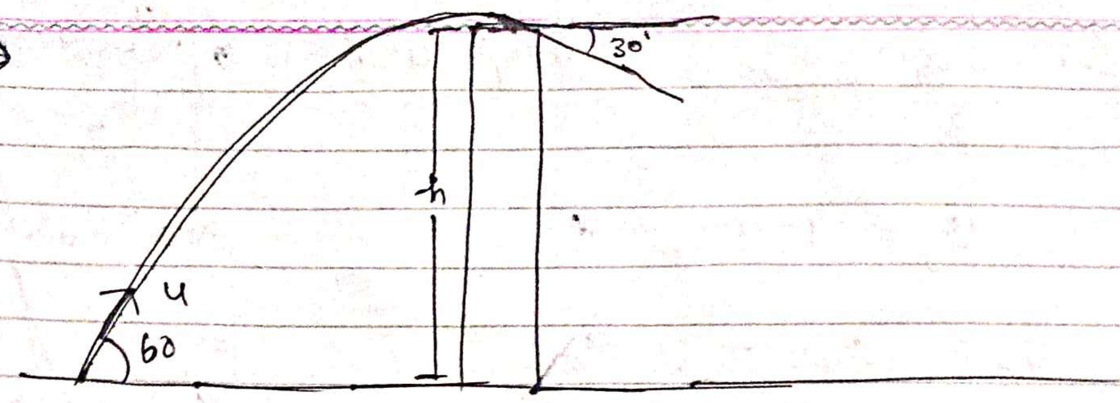
$$t = \frac{4\sqrt{3} - 3}{4}$$

$$= \frac{6.8 - 3}{4}$$

$$\Rightarrow \frac{3.8}{4} \Rightarrow 0.9\ s.$$

similarly we find second time $\phi = 323^\circ$ Ans.

Ques. →



find the height of the tower.

$v \cos 30° = u \cos 60°$ (horizontal velocity is same)

$\Rightarrow \frac{\sqrt{3}v}{2} = \frac{u}{2}$

$v = \frac{u}{\sqrt{3}}$

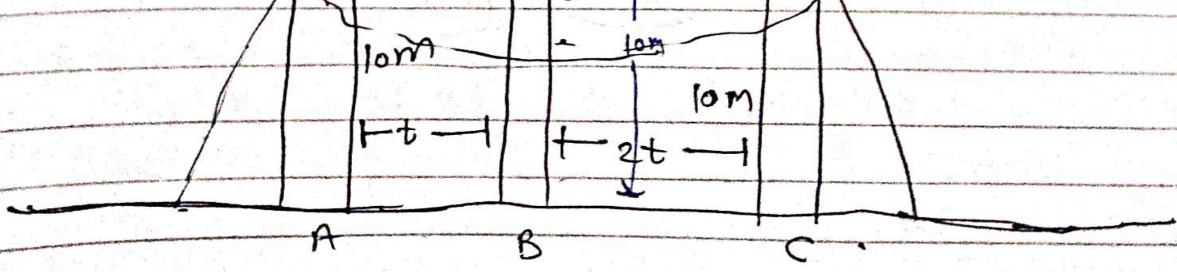
$v^2 - u^2 = 2gh$

$\frac{u^2}{3} - u^2 = -2gh$

$\frac{2u^2}{3} = 2gh$

$h = \frac{u^2}{3g}$ Ans.

Ques.

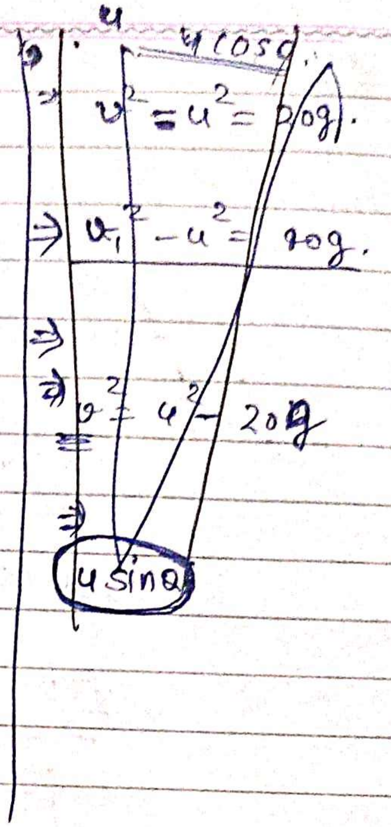


find the max. height above ground surface attain by the particle.

Solve.

$$\frac{2v_y}{g} = 3t$$

$(v_y = u\sin\theta)$



$$v_y = 15t$$

$$s_y = v_y t + \frac{1}{2} a_y t^2$$

$$10 = v_y t + \frac{1}{2}(-10)t^2$$

$$10 = v_y t - 5t^2 \quad \text{—— (ii)}$$

$$10 = 15t^2 - 5t^2$$

$$10 = 10t^2$$

$$t = 1\,\text{sec}$$

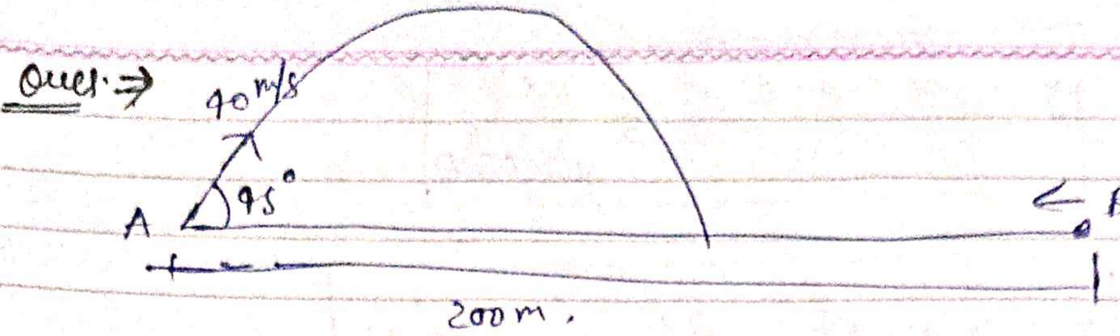
$$v_y = 15\ \text{m/s}$$

$$h = \frac{v_y^2}{20} = \frac{225}{20} = \frac{45}{4}\ \text{m}$$

$$H = 10 + \frac{45}{4} = \frac{85}{4}\ \text{m}$$

Ans.

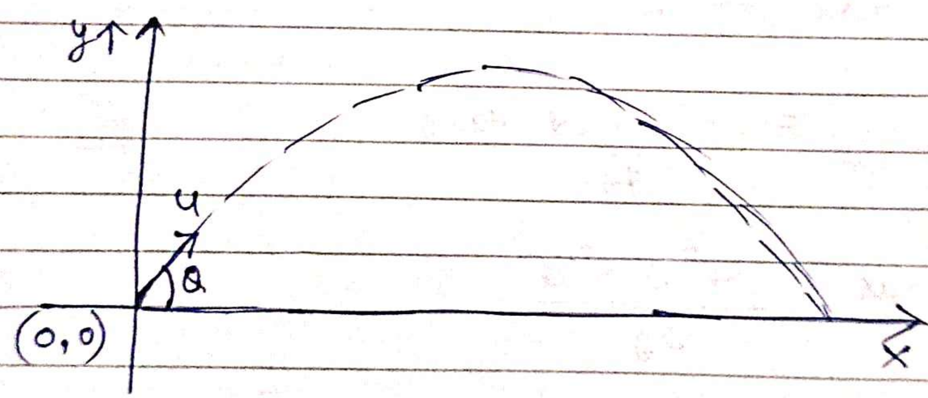
Ques. ⇒



$\Rightarrow \quad 200 = \frac{40 \times 40}{10} \Rightarrow$ (160 m) $\Rightarrow$ (40 m)

$\Rightarrow \quad T = \frac{2 \times 40 \times}{\sqrt{2} \times 10} \Rightarrow (4\sqrt{2})\,s \Rightarrow \frac{40}{4\sqrt{2}} \Rightarrow 5\sqrt{2}$ m/s

* Equation of trajectory (path of particle)



$$x = u\cos\theta\, t$$

$$y = u\sin\theta\, t \pm \frac{1}{2} g t^2$$

$$y = u\sin\theta \left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}\frac{g x^2}{u^2\cos^2\theta}$$

$$y = x \tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta} \quad \text{—— (i)}$$

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2}(1+\tan^2\theta)$$

$$y = x\tan\theta\left(1 - \frac{g\,x}{2u^2\cos^2\theta\tan\theta}\right)$$ from eqn (i)

$$y = x\tan\theta\left(1 - \frac{x}{R}\right)$$

$$y_{max} = \frac{R}{2}\tan\theta \times \frac{1}{2} \qquad \text{when } x = \frac{R}{2}$$

$$= \frac{u^2 \sin 2\theta \tan\theta}{4g}$$

$$y_{max} = \frac{u^2\sin^2\theta}{2g}$$

Ques. $y = ax - bx^2$

find (i) range (ii) max. height

(iii) Time of flight

Solve: (i) $y = 0$

$ax - bx^2 = 0$

$x = 0, \quad x = a/b$

$\Rightarrow R = a/b$

(ii) for y_{max}, $x = \frac{R}{2} = \frac{a}{2b}$

$y_{max} = \frac{a \cdot a}{2b} - \frac{ba^2}{4b^2}$

$\Rightarrow \frac{a^2}{2b} - \frac{a^2}{4b}$

$$y_{max} = \frac{a^2}{4b} = H$$

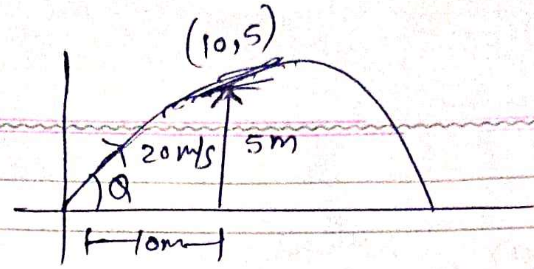
(iii)

$\frac{v_y^2}{2g} = \frac{a^2}{4b}$

$v_y^2 = \frac{a^2 g}{2b} \Rightarrow$

$v_y = a\sqrt{\frac{g}{2b}}$

Ques.



$\theta = ?$

Sol.

~~$v^2 = u^2 - 2gh$~~

~~$v^2 = 400 - 2 \times 10 \times 5$~~

~~$v^2 = 400 - 100$~~

~~$v^2 = 300$~~

~~$v^2 = v_y^2 + v_x^2$~~

~~$300 = 20\, u^2 \sin^2\theta +$~~

$$y = x \tan\theta - \frac{1}{2} \frac{g x^2}{u^2 \cos^2\theta}$$

$$5 = 10 \tan\theta - \frac{1}{2} \times \frac{10}{400} \times 100(1+\tan^2\theta)$$

$$5 = 10 \tan\theta - \frac{5}{4}(1 + \tan^2\theta)$$

$$20 = 40 \tan\theta - 5(1 + \tan^2\theta)$$

$$\tan^2\theta - 8 \tan\theta + 5 = 0$$

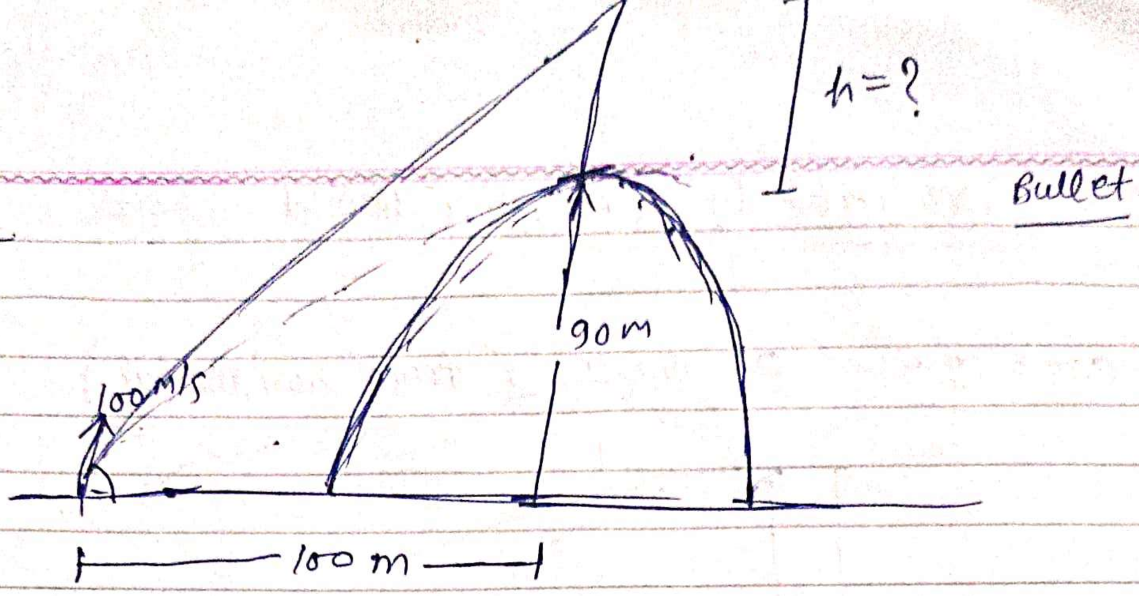
$$\tan\theta = \frac{8 \pm \sqrt{64 - 4(-15)}}{2}$$

$$\tan\theta = \frac{8 \pm 2\sqrt{11}}{2}$$

$$\tan\theta = 4 \pm \sqrt{11}$$

$$\theta = \tan^{-1}(4 - \sqrt{11})$$

$$\theta = \tan^{-1}(4 + \sqrt{11})$$



Ans.

~~⇒ $v^2 = 100\times100 - 2\times10\times90$~~

~~⇒ $10000 - 1800$~~

~~$v^2 \Rightarrow 8200$~~

$$y = x\tan\theta - \frac{1}{2}\frac{g x^2}{u^2\cos^2\theta}$$

$$\Rightarrow 90 = 100\tan\theta - \frac{\frac{1}{2}\times10\times100\times100\,(1+\tan^2\theta)}{100\times100}$$

$$18 = 2\tan\theta - (1+\tan^2\theta)$$

$$\tan^2\theta - 20\tan\theta + 19 = 0$$

$$(\tan\theta - 1)(\tan\theta - 19) = 0$$

$$\theta = 45^0 \quad \text{or} \quad \tan^{-1}(19.)$$

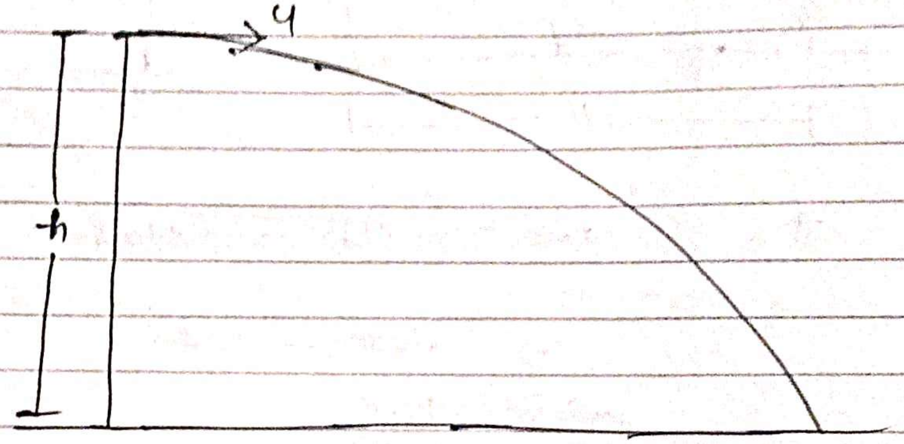
$h = 10$ m

or

$h = 1810$ m Ans.

* Projectile from a height

case I · at $\theta = 0$ (from horizontal)



x - axis. | y - axis.

$U_x = u$ m/s | $U_y = 0$ m/s

$a_x = 0$ | $a_y = g \downarrow$ m/s^2.

* in 'T' time $S_y = h \downarrow$

$$S_y = U_y t + \frac{1}{2} a_y t^2$$

$$h = (0)t + \frac{1}{2} \times g t^2$$

$$t^2 = \frac{2h}{g}$$

$$t = \sqrt{\frac{2h}{g}}$$

it is a interesting point which are independent to horizontal velocity u. and always same. like as motion in free fall (release cond^n).

* Range = S_x in 'T' time.

$= u_x T$

$$R = u\sqrt{\frac{2h}{g}}$$

* velocity at the hit the ground.

$v_y^2 = u_y^2 + 2a_y S_y$

$= (0)^2 + 2gh$

$v_y^2 = 2gh$

$v^2 = v_y^2 + v_x^2$

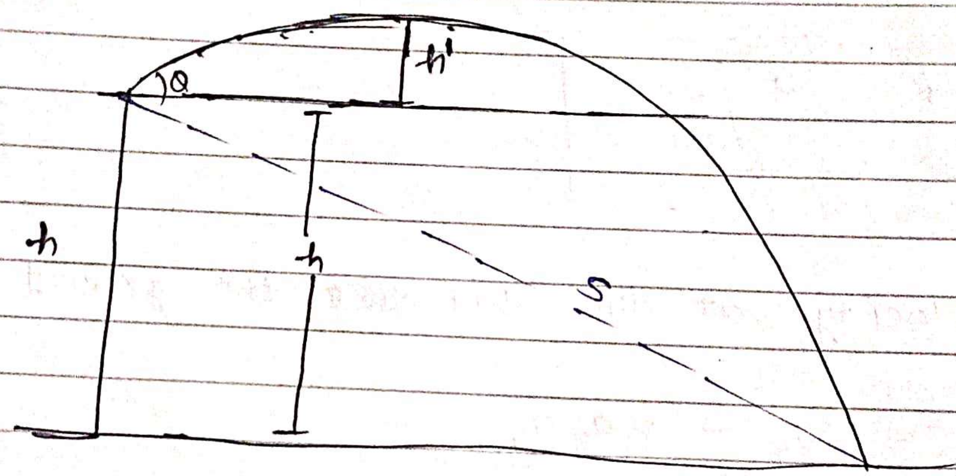
$v^2 = 2gh + u^2$

$$v = \sqrt{u^2 + 2gh}$$

* $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$ θ from the horizontal.

$$\theta = \tan^{-1}\left(\frac{\sqrt{2gh}}{u}\right)$$

Case II: Oblique projection from height



* Time (T)

$$S_y = U_y t + \frac{1}{2} a_y t^2$$

$$-h = u \sin\theta\, T + \frac{1}{2}(-g) T^2$$

* max. heigh (vertical)

$$H = h' + h$$

$$H = \frac{u^2 \sin^2\theta}{2g} + h$$

* Range

$$R = u\cos\theta\, T$$

* velocity (at the hit the ground)

$$v^2 = v_x^2 + v_y^2$$

$$= u^2 \cos^2\theta + (u_y^2 + 2a_y S_y)$$

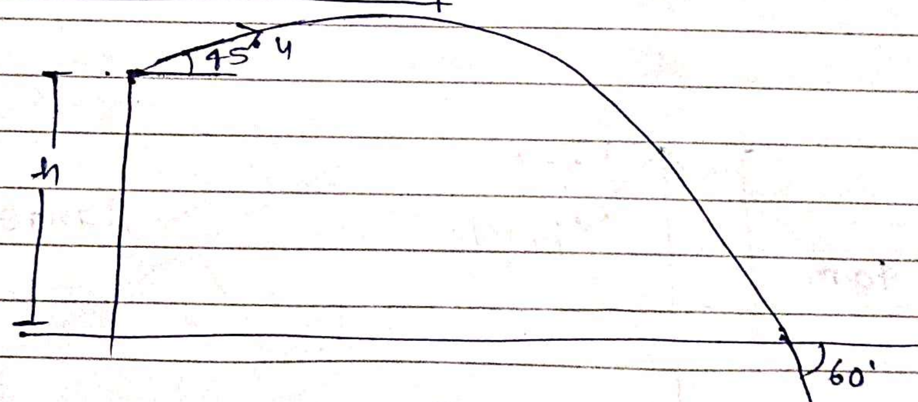
$$= u^2 \cos^2\theta + (u^2 \sin^2\theta + 2(-g)(-h))$$

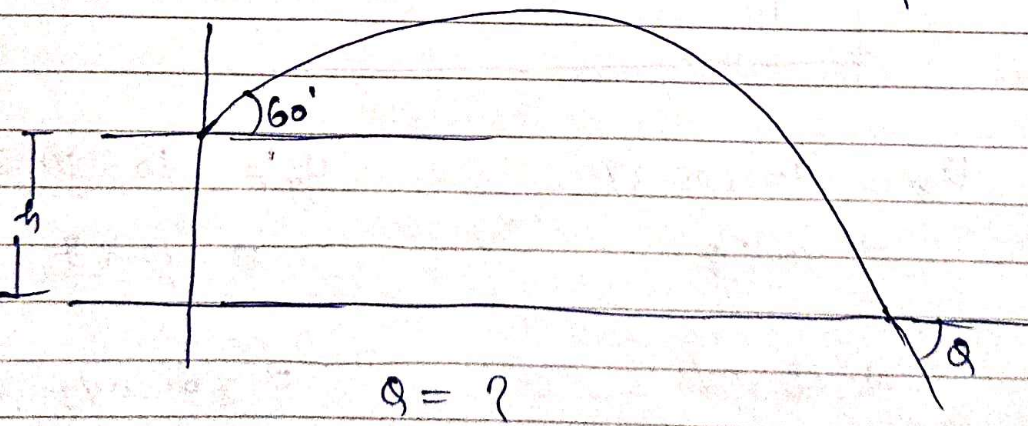
$$v^2 = u^2 \cos^2\theta + u^2 \sin^2\theta + 2gh$$

$$v^2 = u^2 + 2gh$$

$$\boxed{v = \sqrt{u^2 + 2gh}}$$

Ques.





$\theta = ?$

Solve.

$$u \cos 45^\circ = v \cos 60^\circ$$

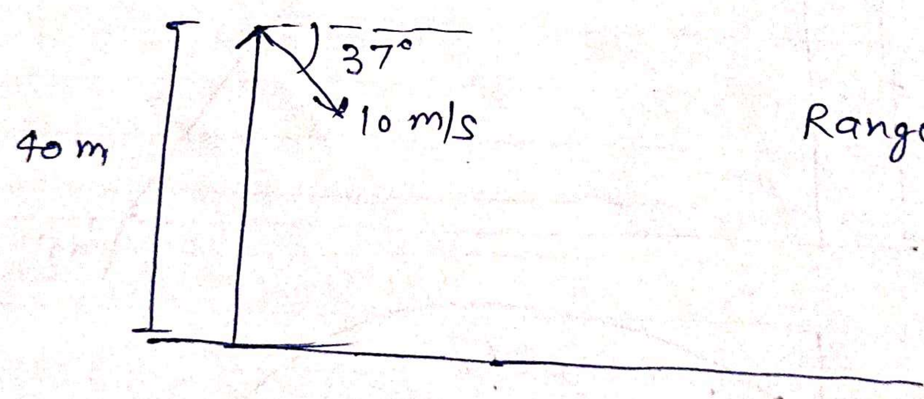
$$u \cos 60^\circ = v \cos \theta$$

$$\frac{1/\sqrt{2}}{1/2} = \frac{1/2}{\cos \theta}$$

$$\cos \theta = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{1}{2\sqrt{2}}\right)}$$ Ans.

Ques.



Range = ?

$v_x = 10 \cos 37$

$\Rightarrow 10 \times \frac{4}{5}$

$\Rightarrow 8$ m/s

$v_y = 10 \sin 37$

$\Rightarrow 10 \times \frac{3}{5}$

$\Rightarrow 6$ m/s.

S in y ⇒

$90 = 6 \times t + \frac{1}{2} \times 10 t^2$

$90 = 6t + 5t^2 - 90$

$T \Rightarrow \frac{-6 \pm \sqrt{36 + 800}}{10}$

⇒ 8(36).

⇒ 9/6

⇒ 2+3

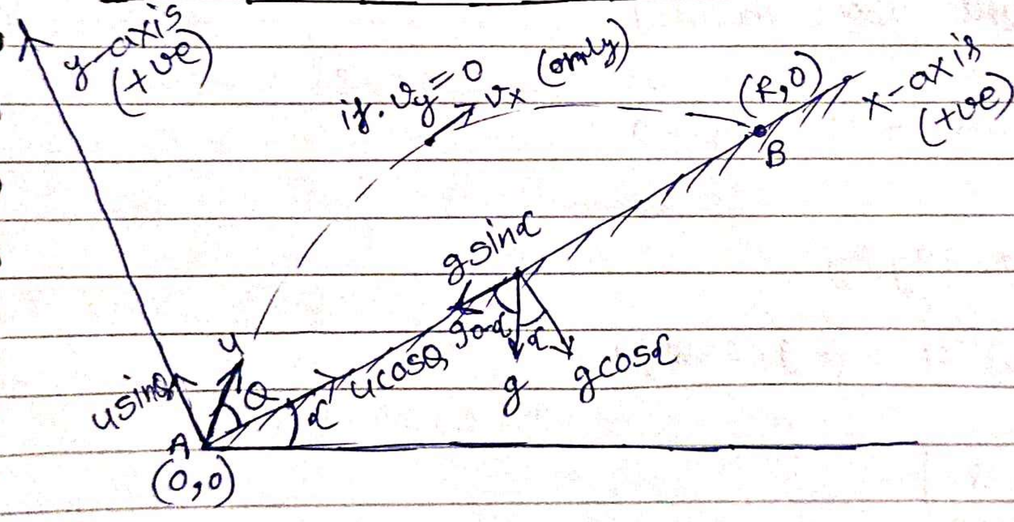
⇒ 26×26

156
52
676

$R = 8 \times T$

Ans.

* Projectile on an incline.



x- direⁿ

$u_x = u \cos \theta$

$a_x = -g \sin \alpha$

y- direⁿ

$u_y = u \sin \theta$

$a_y = -g \cos \alpha$

* in time $s_y = 0$

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$$0 = (u\sin\theta)T + \frac{1}{2}(-g)\cos\alpha\, T^2$$

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

$$\boxed{T = \frac{2u_y}{|a_y|}}$$

* maximum height above inclined

$H = s_y$ by the time $v_y = 0$

$$v_y^2 = u_y^2 + 2a_y s_y$$

$$(0)^2 = (u\sin\theta)^2 + 2(-g\cos\alpha)H$$

$$\boxed{H = \frac{u^2\sin^2\theta}{2g\cos\alpha}}$$

* time to reach max. height above inclined

$$v_y = u_y + a_y t$$

$$0 = u\sin\theta + (-g)\cos\alpha\, t_a$$

$$\boxed{t_a = \frac{u\sin\theta}{g\cos\alpha}}$$

* Range

$R = S_x$ in 'T' time

$$R = u\cos\theta\, T + \frac{1}{2}(-g\sin\alpha)T^2$$

$$R = u\cos\theta \cdot \frac{2u\sin\theta}{g\cos\alpha} - \frac{g\sin\alpha}{2}\,\frac{4u^2\sin^2\theta}{g^2\cos^2\alpha}$$

$$R = \frac{2u^2\sin\theta}{g\cos\alpha}\left\{\cos\theta - \frac{\sin\alpha\sin\theta}{\cos\alpha}\right\}$$

$$R = \frac{2u^2\sin\theta}{g\cos\alpha}\left\{\frac{\cos(\theta+\alpha)}{\cos\alpha}\right\}$$

$$\boxed{R = \frac{2u^2\sin\theta\cos(\theta+\alpha)}{g\cos^2\alpha}}$$

* max. Range.

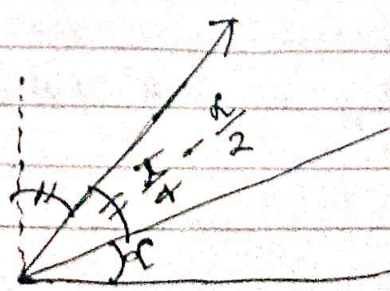
$$\frac{d}{d\theta}\left(\sin\theta\cos(\theta+\alpha)\right) = 0$$

$$-\sin\theta\sin(\theta+\alpha) + \cos(\theta+\alpha)\cos\theta = 0$$

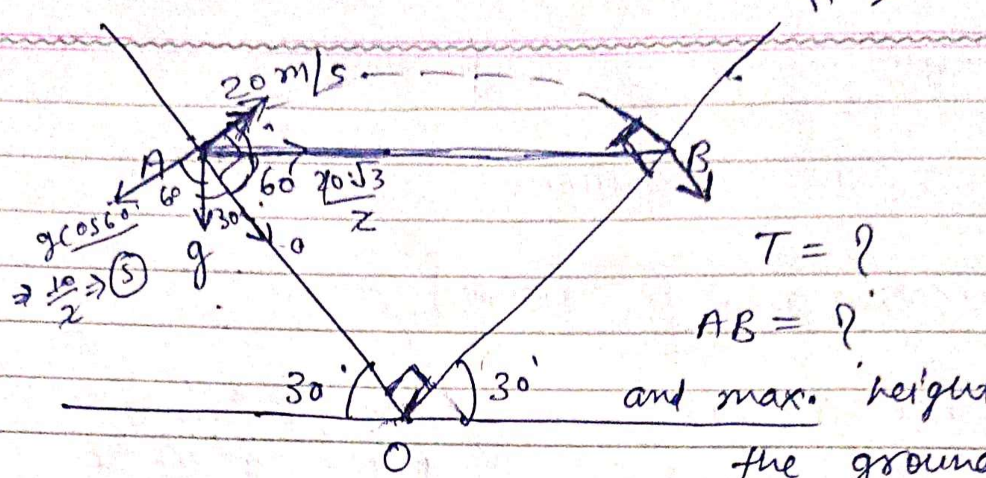
$$\cos(2\theta+\alpha) = 0$$

$$2\theta + \alpha = \frac{\pi}{2}$$

$$\boxed{\theta = \frac{\pi}{4} - \frac{\alpha}{2}}$$



Ques.



T = ?

AB = ?

and max. height from the ground surface

Sol. ⇒

$v_x = u_x + a_x t$

$0 = 20 - 5t$

$t = 4\,s$

$AB = \sqrt{OA^2 + OB^2}$

$\Rightarrow 20 \times 4 + \frac{1}{2} \times (-5) \times 16$

$\Rightarrow 80 - 40$

$\Rightarrow 40\,m$ Ans.

$OA = 40\sqrt{3}$

$H = \frac{(20)^2 \sin^2 30}{2g} = 5\,m$

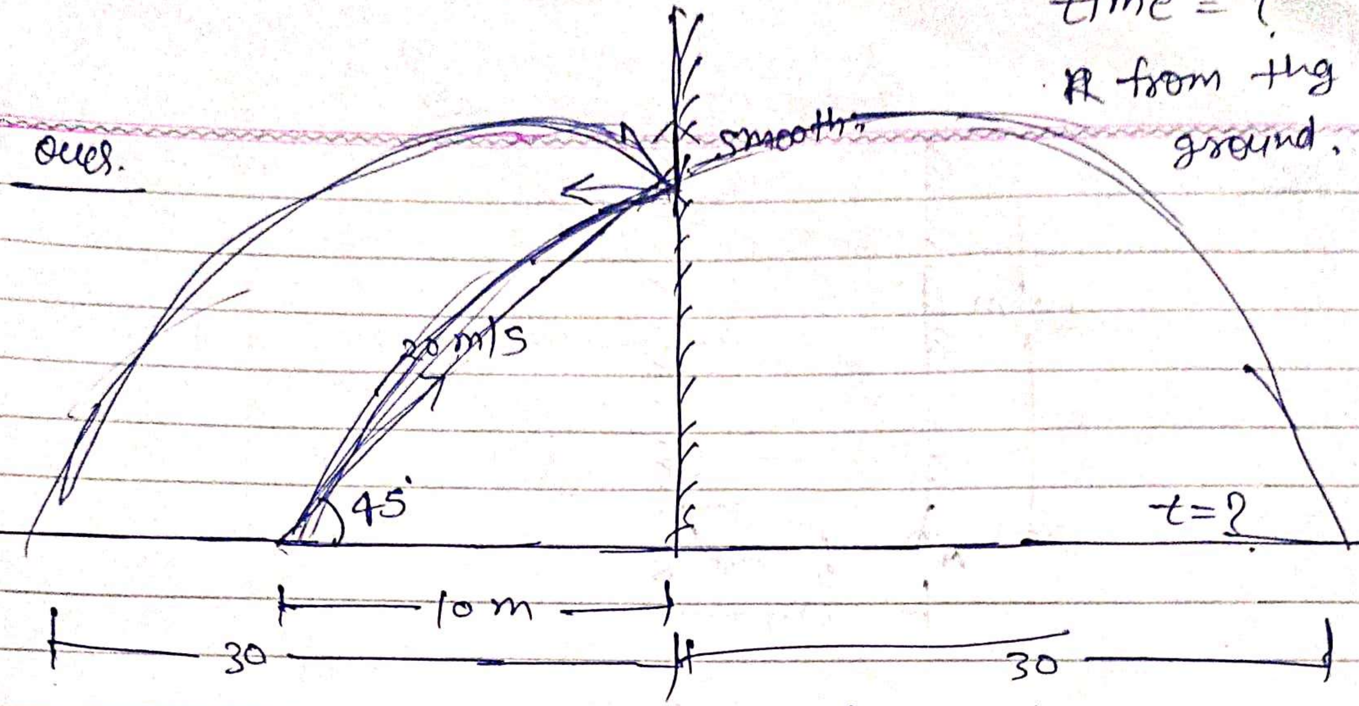
$h_A = OA \sin 60 = 40\sqrt{3} \times \frac{\sqrt{3}}{2}$

$\Rightarrow 60\,m$

$H = 65\,m$

Ques.

time = ?
R from the ground.



particles goes elastic collision with the walls.

⇒ ~~$u^2 = 2gh$~~

$$y = 10 \times 1 - \frac{1 \times 10 \times 10 \times 10 \times 2}{2 \times 20 \times 20 \times 1}$$

$$y = 10 - \frac{5}{2}$$

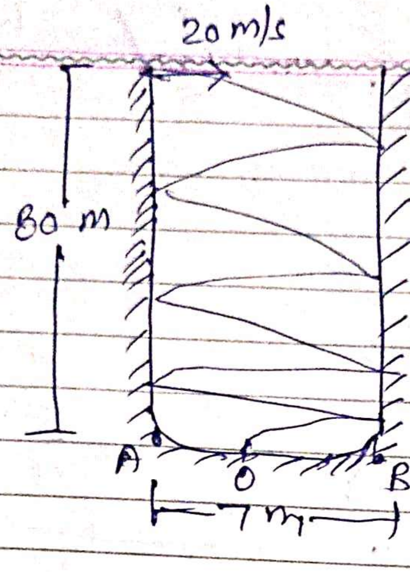
$$y = \frac{15}{2}$$

⇒ $R = \frac{u^2 \sin 2\theta}{g}$

⇒ $20 \times 20 \times$

⇒

Ques.



⇒ from A = 4m.

⇒ from B = 3m.