If 0 < a, b < 1, and $\tan^{-1}a + \tan^{-1}b = \pi/4$, then the value of

$(a+b) - (\frac{a^2+b^2}{2}) + (\frac{a^3+b^3}{3}) - (\frac{a^4+b^4}{4}) + \dots$ is

(1) $log_e 2$

(2) $e^2 - 1$

(3) e

(4) $log_e(\frac{e}{2})$

Correct option is (1) $log_e 2$

$\tan^{-1}a + \tan^{-1}b = \pi/4 \quad 0 < a, b < 1$

$\Rightarrow \frac{a+b}{1-ab} = 1$

a + b = 1 - ab

(a + 1)(b + 1) = 2

Now $[a - \frac{a^2}{2} + \frac{a^3}{3} + \dots] + [b - \frac{b^2}{2} + \frac{b^3}{3} + \dots]$

$= log_e(1+a) + log_e(1+b)$

(∵ expansion of $\log_e(1 + x)$)

$= \log_e[(1 + a)(1 + x)]$

$= \log_e 2$