

The number of solutions of the equation

$$\sin^{-1}\left[x^2 + \frac{1}{3}\right] + \cos^{-1}\left[x^2 - \frac{2}{3}\right] = x^2,$$

for $x \in [-1, 1]$, and $[x]$ denotes the greatest integer less than or equal to x , is :

- (1) 2
- (2) 0
- (3) 4
- (4) Infinite

Given equation

$$\sin^{-1}\left[x^2 + \frac{1}{3}\right] + \cos^{-1}\left[x^2 - \frac{2}{3}\right] = x^2$$

Now, $\sin^{-1}\left[x^2 + \frac{1}{3}\right]$ is defined if

$$-1 \leq x^2 + \frac{1}{3} < 2 \Rightarrow \frac{-4}{3} \leq x^2 < \frac{5}{3}$$

$$\Rightarrow \boxed{0 \leq x^2 < \frac{5}{3}} \quad \dots(1)$$

and $\cos^{-1}\left[x^2 - \frac{2}{3}\right]$ is defined if

$$-1 \leq x^2 - \frac{2}{3} < 2 \Rightarrow \frac{-1}{3} \leq x^2 < \frac{8}{3}$$

$$\Rightarrow \boxed{0 \leq x^2 < \frac{8}{3}} \quad \dots(2)$$

So, from (1) and (2) we can conclude

$$0 \leq x^2 < \frac{5}{3}$$

Case - I if $0 \leq x^2 < \frac{2}{3}$

$$\sin^{-1}(0) + \cos^{-1}(-1) = x^2$$

$$\Rightarrow x + \pi = x^2$$

$$\Rightarrow x^2 = \pi$$

but $\pi \notin \left[0, \frac{2}{3}\right)$

\Rightarrow No value of 'x'

Case - II if $\frac{2}{3} \leq x^2 < \frac{5}{3}$

$$\sin^{-1}(1) + \cos^{-1}(0) = x^2$$

$$\Rightarrow \frac{\pi}{2} + \frac{\pi}{2} = x^2$$

$$\Rightarrow x^2 = \pi$$

but $\pi \notin \left[\frac{2}{3}, \frac{5}{3}\right)$

\Rightarrow No value of 'x'

So, number of solutions of the equation is zero.