

The sum of possible values of x for

$$\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\left(\frac{8}{31}\right) \text{ is :}$$

A $-\frac{32}{4}$

B $-\frac{33}{4}$

C $-\frac{31}{4}$

D $-\frac{30}{4}$

$$\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\left(\frac{8}{31}\right)$$

$$\Rightarrow \tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$$

$$\Rightarrow \tan^{-1} \left(\frac{(x+1)+(x-1)}{1-(x+1)(x-1)} \right) = \tan^{-1}\left(\frac{8}{31}\right)$$

$$\Rightarrow \frac{(1+x)+(x-1)}{1-(1+x)(x-1)} = \frac{8}{31}$$

$$\Rightarrow \frac{2x}{2-x^2} = \frac{8}{31}$$

$$\Rightarrow 4x^2 + 31x - 8 = 0$$

$$\Rightarrow x = -8, \frac{1}{4}$$

but at $x = \frac{1}{4}$

$LHS > \frac{\pi}{2}$ and $RHS < \frac{\pi}{2}$

So, only solution is $x = -8 = -\frac{32}{4}$