

Past Year JEE Questions

Questions

Question: 01

Let PQ be a diameter of the circle $x^2 + y^2 = 9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, $x + y = 2$ respectively, then the maximum value of $\alpha\beta$ is ____.

Solutions

Solution: 01

Answer

Correct Answer is 7

Explanation

Let $P(3 \cos \theta, 3 \sin \theta)$

$Q(-3 \cos \theta, -3 \sin \theta)$

$$\alpha = \left| \frac{3 \cos \theta + 3 \sin \theta - 2}{\sqrt{2}} \right|$$

$$\beta = \left| \frac{-3 \cos \theta - 3 \sin \theta - 2}{\sqrt{2}} \right|$$

$$\alpha\beta = \left| \frac{(3 \cos \theta + 3 \sin \theta - 2)(-3 \cos \theta - 3 \sin \theta - 2)}{2} \right|$$

$$= \left| \frac{5+9 \sin 2\theta}{2} \right|^2$$

$$\alpha\beta_{\max} = \frac{5+9}{2} = 7 \text{ (when } \sin 2\theta = 1\text{)}$$