27. Find the equation of a circle passing through the point (7, 3) having radius 3 units and whose centre lies on the line y = x – 1.

Solution:

Since, the equation of a circle having centre (h, k), having radius as r units, is

$(x – h)^2 + (y – k)^2 = r^2$Centre lies on the line i.e., y = x – 1,

Co – Ordinates are (h, k) = (h, h – 1)

$(x – h)^2 + (y – k)^2 = r^2$

$(7 – h)^2 + (3 – (h – 1))^2 = 3^2$

$49 + h^2 – 14h + (3 – h +1)^2 = 9$

On rearranging we get

$h^2 – 14h + 49 +16 +h^2 – 8h – 9 = 0$

$2h^2 – 22h + 56 = 0$

$h^2 – 11h + 28 = 0$

$h^2 – 4h – 7h + 28 = 0$

h (h – 4) – 7 (h – 4) = 0

(h – 7) (h – 4) = 0

h = 7 or 4

Centre = (7, 6) or (4, 3)

$(x – h)^2 + (y – k)^2 = r^2$

Equation, having centre (7, 6)

$(x – 7)^2 + (y – 6)^2 = 3^2$

$x^2 – 14x + 49 + y^2 – 12y + 36 – 9 = 0$

$x^2 – 14x + y^2 – 12y + 76 = 0$

Equation, having centre (4, 3)

$(x – 4)^2 + (y – 3)^2 = 3^2$

$x^2 – 8x + 16 + y^2 – 6y + 9 – 9 = 0$

$x^2 – 8x + y^2 – 6y + 16 = 0$

Hence, the required equation is $x^2 – 14x + y^2 – 12y + 76 = 0$ or $x^2 – 8x + y^2 – 6y + 16 = 0$.

If your circles basics is stable, then it is a quite simple problem.
Tip: try to remember important formulas given in the tips and tricks section.