

Que-4: Find the value of $\sum_{i=0}^{\infty} \frac{i^2}{i!}$

$$\text{Sol: } \sum_{i=0}^{\infty} \frac{i^2}{i!} = \sum_{i=0}^{\infty} \frac{i^2}{i \times (i-1)!} = \sum_{i=1}^{\infty} \frac{i}{(i-1)!} = \sum_{i=1}^{\infty} \frac{i-1+1}{(i-1)!} = \sum_{i=1}^{\infty} \frac{1}{(i-1)!} + \sum_{i=2}^{\infty} \frac{1}{(i-2)!} = 2e$$

Que-5: Find the coefficient of x^4 in $(1 + 2x + 3x^2)e^{-x}$

Ans: $(1 + 2x + 3x^2)e^{-x}$

$$= (1 + 2x + 3x^2) \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \dots \right)$$

So, coefficient of $x^4 = -\frac{2}{3!} + \frac{3}{2!} + \frac{1}{4!}$

$$\begin{aligned} &= \frac{1}{24} + \frac{3}{2} - \frac{2}{6} \\ &= \frac{1 - 8 + 36}{24} \\ &= \frac{29}{24} \end{aligned}$$