

Que-1: Find the value of $1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \dots$

Ans: We can see the k^{th} term as

$$\begin{aligned}\frac{\sum_1^k i}{k!} &= \frac{\left\{ \frac{k(k+1)}{2} \right\}}{k!} \\ &= \frac{1}{2} \left\{ \frac{k+1}{(k-1)!} \right\} \\ &= \frac{1}{2} \left\{ \frac{k-1+2}{(k-1)!} \right\} \\ &= \frac{1}{2} \left\{ \frac{1}{(k-2)!} + \frac{2}{(k-1)!} \right\}\end{aligned}$$

$$\begin{aligned}\text{So, now } 1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \dots &= \sum_{k=0}^{\infty} \frac{1}{2} \left\{ \frac{1}{(k-2)!} + \frac{2}{(k-1)!} \right\} \\ &= \frac{1}{2} \left\{ \sum_{k=2}^{\infty} \frac{1}{(k-2)!} + \sum_{k=1}^{\infty} \frac{2}{(k-1)!} \right\} \\ &= \frac{1}{2} \{e + 2e\} \\ &= \frac{3}{2}e\end{aligned}$$